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STABILITY AND INSTABILITY OF SOLITARY WAVES FOR ONE-DIMENSIONAL SINGULAR SCHRÖDINGER EQUATIONS

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Abstract. In this paper, we investigate the stability properties of the standing wave $u = \phi_\omega(x)e^{i\omega t}$, $x \in \mathbb{R}$, $t \geq 0$ for a class of singular Schrödinger equations. By the use of spectral decomposition technique, we prove that u is stable (unstable) if the $L_2(\mathbb{R})$-norm $\|\phi_\omega\|$ increases (decreases) with ω.

Introduction. This paper is concerned with the stability and instability properties of solitary wave solutions of a general class of Schrödinger equations of the form

$$\begin{aligned} &iu_t = -u_{xx} + u[f(|u|^2) + 2kh'(|u|^2)(h(|u|^2))_{xx}] \\ &u(x,0) = u_0(x), \quad x \in \mathbb{R}; \quad 0 \leq t < T; \quad k \in \mathbb{R}. \end{aligned} \tag{0.1}$$

A solitary wave is a localized, finite energy solution of a nonlinear evolution equation. Fundamental equations in the theory of nonlinear waves that possess such solutions are the nonlinear Schrödinger equations (NLS). NLS appears in various problems, modeling many phenomena such as the behaviour of non-ideal Bozegas with a weak particle interaction, the spreading of the heat impulse in solids, the Langmuir waves in plasma, etc. (see [1], [14], [15]).

Some discussion of linearized stability for NLS with a cubic nonlinearity ($f(s) = -s$, $h(s) = 0$) appeared in [16]. The first rigorous result concerning the nonlinear stability of standing waves for NLS with a logarithmic nonlinearity $f(s) = -\log s$, $h(s) = 0$ was obtained by T. Cazenave [5]. In the paper [6], via use of the concentration-compactness principle introduced by P.L. Lions [11], T. Cazenave and P.L. Lions proved the orbital stability of a standing wave for NLS in case $f(s) = -s^p$ $(0 < p < 2)$, $h(s) = 0$. In [17], [18], developing the ideas advanced by T. Benjamin [2], [3] we have independently proved the orbital stability of a solitary wave for NLS in case $f(s) = \lambda - s^p$ $(0 < 2p \leq 3)$, $h(s) = 0$. In [13] the stability properties of standing waves for NLS with $h(s) = 0$ and general nonlinearity $f(s)$ were obtained by M. Weinstein.

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