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LINEARIZED OSCILLATIONS IN NONAUTONOMOUS DELAY DIFFERENTIAL EQUATIONS

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Abstract. Our aim in this paper is to establish linearized oscillation results for quite general nonlinear and nonautonomous equations of the form

$$\dot{x}(t) + f(t, x(t - \tau_1(t)), \dots, x(t - \tau_m(t))) = 0.$$

Applications to various equations in mathematical biology are also given.

1. Introduction. Recently, a linearized oscillation theory has been developed in [6–8] and [3–5] according to which the study of the oscillation of certain nonlinear autonomous delay equations can be reduced to the study of the oscillation of an associated linear equation with constant coefficients.

Our aim in this paper is to establish linearized oscillation results for quite general nonlinear and nonautonomous equations of the form

$$\dot{x}(t) + f(t, x(t - \tau_1(t)), \dots, x(t - \tau_m(t))) = 0, \tag{1}$$

where

$$\begin{cases} f \in C[[t_0, \infty) \times \mathbb{R}^m, \mathbb{R}],\ f(t, u_1, \dots, u_m) \geq 0 \text{ for } u_1, \dots, u_m \geq 0, \\ f(t, u_1, \dots, u_m) \leq 0 \text{ for } u_1, \dots, u_m \leq 0, \text{ and for } i = 1, \dots, m, \\ \tau_i \in C[[t_0, \infty), (0, \infty)] \text{ and } \lim_{t \to \infty} [t - \tau_i(t)] = \infty. \end{cases} \tag{2}$$

We show that, under appropriate hypotheses, the oscillatory behavior of equation (1) is characterized by the oscillatory behavior of an associated linear differential equation with variable coefficients of the form

$$\dot{y}(t) + \sum_{i=1}^{m} p_i(t) y(t - \tau_i(t)) = 0, \tag{3}$$

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