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UNIQUENESS OF SOLUTIONS OF NONLINEAR DIRICHLET PROBLEMS

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(Submitted by: H. Brezis)

Abstract. In this paper we prove the uniqueness of solution for the problem

$$\begin{aligned} -\Delta u &= u^p + \lambda u \quad && \text{in } \Omega \\ u &> 0 && \text{in } \Omega \\ u &= 0 && \text{on } \partial\Omega, \end{aligned}$$

where $\Omega \subset \mathbb{R}^n$ is the unit ball with $n \geq 3$, $1 < p \leq \frac{n+2}{n-2}$ and $\lambda > 0$.

Introduction. In this paper we consider the problem

$$\left.\begin{aligned} -\Delta u &= u^p + \lambda u \quad && \text{in } \Omega \\ u &> 0 && \text{in } \Omega \\ u &= 0 && \text{on } \partial\Omega, \end{aligned}\right\} \qquad (\mathrm{P}_\lambda)$$

where $\Omega = B(0,1) = \{x \in \mathbb{R}^n : |x| < 1\}$, $n \geq 3$, $1 < p \leq \frac{n+2}{n-2}$, $\lambda > 0$ and prove the uniqueness of the solution whenever existence of such a solution is known. Previous to this work, the question of uniqueness had been considered in [5] and the solution shown to be unique for p satisfying $1 < p \leq \frac{n}{n-2}$. In [3], uniqueness is proved for $\lambda < 0$ and $1 < p < \frac{n+2}{n-2}$.

Even if we strongly exploit the fact that solutions are radial ([2]) as in [3, 4] our approach is different from both [3, 4]. Also our proof shows how Pohozaev's identity [5] enters even the question of uniqueness. In the next section we state and prove our main result. As already noted, since in our situation all solutions of P_λ are radial by [2], throughout the discussion we work in the space of $C^2_R(\Omega)$, the space of twice continuously differentiable radial functions.

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