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A BOUNDARY VALUE PROBLEM WHOSE JUMPING NONLINEARITY IS NEITHER SMOOTH NOR LIPSCHITZIAN

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Abstract. Let Ω be a bounded domain in $\mathbb{R}^N (N \geq 1)$ with smooth boundary $\partial\Omega$ and $g : \mathbb{R} \to \mathbb{R}$ for which $\lim_{s\to\pm\infty} \frac{g(s)}{s}$ exist. For a given smooth function h_1 on Ω and a smooth function φ positive on Ω, we are concerned with the number of solutions for large t of the problem $-\Delta u = g(u) + t\varphi + h_1$ on Ω, $u = 0$ on $\partial\Omega$. We shall assume only that $g(\cdot)$ is continuous, in contrast with many other works which require $g(\cdot)$ to be continuously differentiable and one-sided Lipschitzian.

I. Introduction. Let $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be a bounded domain with smooth boundary $\partial\Omega$. We denote by

$$0 < \lambda_1 < \lambda_2 < \cdots < \lambda_k < \cdots$$

the sequence of distinct eigenvalues of the eigenvalue problem

$$-\Delta u = \lambda u \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega. \tag{1}$$

Let φ_1 be the eigenfunction corresponding to λ_1 with $\varphi_1 > 0$ on Ω and $\int_\Omega \varphi_1^2\, ds = 1$.
The following theorem is proved by Lazer and McKenna in [9], Theorem 2.4.

Theorem 2.4 of [9]. *Suppose that $g(\cdot) : \mathbb{R} \to \mathbb{R}$ is of class C^1, that for some $k > 1$ and some $b_1 < \lambda_{k+1}$,*

$$g'(s) \leq b_1 \quad \textit{for each} \quad s \in \mathbb{R}. \tag{2}$$

Then for each $b \in (\lambda_k, \lambda_{k+1})$ there exists $a^*(b) \in (\lambda_{k-1}, \lambda_k)$ with the property that if

$$\lim_{s\to-\infty} g'(s) = a \in (a^*(b), \lambda_k), \quad \lim_{s\to+\infty} g'(s) = b, \tag{3}$$

then for every smooth h_1, $h_1 \perp \varphi_1$ in $L^2(\Omega)$, there exists $t_0 > 0$ such that when $t \geq t_0$ the boundary value problem (abbreviated to BVP in the sequel)

$$-\Delta u = g(u) + t\varphi_1 + h_1 \textit{ in } \Omega, \quad u = 0 \textit{ on } \partial\Omega \tag{4}$$

has at least three solutions.

It seems to us that three questions might come to mind concerning Theorem 2.4 of [9].

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