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A CLASS OF NONLINEAR PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we deal with a class of nonlinear integro-differential equations in the form of $u_t - u_{xx} = a\frac{d}{dt}\max(\int_0^1 u(x,t)dx, 0) + f(x,t)$, which arise from a one-dimensional quasistatic contact problem in thermoelasticity. The questions of existence, uniqueness and regularity are studied for this kind of problem with related boundary and initial conditions in the case of $a < 1$ and examples of nonexistence and nonuniqueness of the problem are given when the parameter $a = 1$.

1. Introduction.

We shall discuss the solvability of a nonlinear parabolic integro-differential equation in the form of

$$u_t - u_{xx} = a\frac{d}{dt}\max\Big(\int_0^1 u(x,t)dx, 0\Big) + f(x,t) \quad \text{in } Q_T = (0,1)\times(0,T), \tag{1.1}$$

where a is a parameter, with relevant boundary and initial conditions.

The problem is motivated by [6], where a similar equation is used to model a one-dimensional quasi-static contact problem in thermoelasiticity with appropriate boundary conditions (For more information about this model, see also [6] and references therein). We shall discuss the wellposedness of the problem. In particular, we are interested in the regularity of a solution since, in general, the right hand side of (1.1) may not necessarily be continuous. So the best regularity of the solution we might expect is in $W_p^{2,1}(Q_T)$ with $p \le \infty$. In this paper, we shall prove the existence of $W_\infty^{2,1}(Q_T)$ solutions for the problem (1.1) with Dirichlet and Neumann boundary conditions respectively. We shall also give sufficient conditions for existence of smooth solutions for the problems with Neumann boundary conditions and examples of nonexistence and nonuniqueness of the problem in the case of $a = 1$.

In Section 2, we state the problem and list some relevant notations and definitions used in this paper. Then, in Section 3, we establish the existence and uniqueness of $W_2^{2,1}(Q_T)$ solutions with Dirichlet and Neumann boundary conditions respectively. We study the regularity of the solution in Section 4. Lastly, in Section 5, we discuss the problem with $a = 1$.

2. The problem and notations.

The present paper will investigate a nonlinear parabolic integro-differential equation

$$u_t - u_{xx} = a\frac{d}{dt}\max\Big(\int_0^1 u(x,t)dx, 0\Big) + f(x,t) \quad \text{in } Q_T = (0,1)\times(0,T), \tag{2.1}$$

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