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STRONG NONSUBORDINACY AND ABSOLUTELY CONTINUOUS SPECTRA FOR STURM-LIOUVILLE EQUATIONS

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Abstract. The Gilbert-Pearson theory of subordinacy is extended to a generalized Sturm-Liouville system. A condition of strong nonsubordinacy is used to prove that the spectral function associated with the Sturm-Liouville system satisfies upper and lower Lipschitz conditions. A class of equations is investigated which satisfies the condition of strong non-subordinacy.

1. Introduction. We consider the location of the absolutely continuous spectra for operators associated with a general Sturm-Liouville system

$$\begin{aligned} R(t)u'(t) &= u^{[1]}(t), \\ u^{[1]}(t) &= u^{[1]}(a) + \int_a^t [dQ(s) + zdW(s)]u(s), \end{aligned} \tag{1.1}$$

where the real functions R, Q, W satisfy on $a \le t < b$:

(i) $R(t) > 0$, $1/R(t)$ is locally Lebesgue integrable,
(ii) Q, W are locally of bounded variation with W non-decreasing,
(iii) $z = \lambda + i\epsilon$ is a complex parameter with $\operatorname{Im} z = \epsilon \ge 0$.

By a solution of (1.1) is meant a function u, absolutely continuous on compact intervals, such that (1.1) holds almost everywhere. Standard existence-uniqueness results apply to (1.1) (cf. [4, 18]); for every pair of numbers (real or complex) $u(a)$, $u^{[1]}(a)$, there is a unique function u, absolutely continuous locally, such that (1.1) holds almost everywhere. The corresponding function $u^{[1]}$ is locally of bounded variation.

In particular, we make use of the solutions $u_1 = u_1(x, z, \alpha)$, $u_2 = u_2(x, z, \alpha)$, $0 \le \alpha < 2\pi$, defined by the initial conditions

$$\begin{pmatrix} u_1 & u_2 \\ u_1^{[1]} & u_2^{[1]} \end{pmatrix} (a, z, \alpha) = \begin{pmatrix} -\sin\alpha & \cos\alpha \\ \cos\alpha & \sin\alpha \end{pmatrix}. \tag{1.2}$$

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