Differential and Integral Equations, Volume 6, Number 3, May 1993, pp. 553–571.

SOME EXISTENCE RESULTS FOR SINGULAR BOUNDARY VALUE PROBLEMS

L.E. BOBISUD, J.E. CALVERT, AND W.D. ROYALTY
Department of Mathematics and Statistics, University of Idaho, Moscow, Idaho 83843

(Submitted by: C. Corduneanu)

Abstract. The topological transversality theorem is used to establish existence of positive solutions to the differential equation $y'' + f(t,y) = 0$ subject either to the boundary conditions $y(0) = y'(1) = 0$ or to $y(0) = y(1) = 0$. f is allowed to be singular at $y = 0$, $t = 0$, or $t = 1$.

Introduction. We study existence of positive solutions to some second-order boundary value problems of the form

$$y''(t) + f(t, y(t)) = 0, \qquad 0 < t < 1 \tag{1}$$

with either the mixed boundary conditions

$$y(0) = 0 = y'(1) \tag{2}$$

or the Dirichlet boundary conditions

$$y(0) = y(1) = 0. \tag{3}$$

The problem may be singular because we allow $f(t,y)$ to be singular at $y = 0$, $t = 0$, and $t = 1$. Such problems have been studied extensively in recent years [1-8, 11-17]. As in many of these references, we here use *a priori* bounds and the topological transversality theorem [9, 10]. Our particular treatment is based on exploiting the observation that if $f(t,y)$ is a decreasing function of y, then close upper bounds on y'', and hence on y', will follow from close lower bounds on y. Consequently, our results typically allow somewhat stronger singularities than do many studies of such singular problems. In particular, we do not uniformly impose the overly restrictive condition that f be integrable in t for fixed $y > 0$.

The mixed problem. Define for suitable functions u and for $p \geq 1$ the norms

$$\|u\|_0 = \max_{[0,1]} |u(x)|, \quad \|u\|_p = \left\{ \int_0^1 |u(x)|^p \, dx \right\}^{1/p}, \quad |||u|||_p = \max\big(\|u\|_0, \|u'\|_p\big).$$

Received February 1992, in revised form July 1992.
AMS Subject Classifications: 34B15.