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UNIFORM BOUNDARY STABILIZATION OF SEMILINEAR WAVE EQUATIONS WITH NONLINEAR BOUNDARY DAMPING

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Abstract. A semilinear model of the wave equation with nonlinear boundary conditions and nonlinear boundary velocity feedback is considered. Under the assumption that the velocity boundary feedback is dissipative and that the other nonlinear terms are conservative, uniform decay rates for the solutions are established.

1. Introduction. Consider the semilinear equation

$$\begin{cases} y_{tt} = \Delta y - f_0(y) & \text{in } \Omega \times (0,\infty), \\ \frac{\partial y}{\partial \gamma} = -g(y_t \,|_{\Gamma_1}) - f_1(y \,|_{\Gamma_1}) & \text{on } \Gamma_1 \times (0,\infty), \\ y = 0 & \text{on } \Gamma_0 \times (0,\infty), \\ y(0) = y_0 \in H^1_{\Gamma_0}(\Omega), \quad y_t(0) = y_1 \in L_2(\Omega). \end{cases} \tag{1.1}$$

Here Ω is a bounded open region in $\mathbb{R}^n$, $n \geq 1$, with a smooth boundary $\Gamma \equiv \Gamma_0 \cup \Gamma_1$ and $H^1_{\Gamma_0}(\Omega) \equiv \{h \in H^1(\Omega) : h\,|_{\Gamma_0} = 0\}$, where Γ_0 and Γ_1 are closed and disjoint; γ is an outer unit vector normal to the boundary Γ_1. The following assumptions are made on the nonlinear functions f_i, $i = 0, 1$, and g:

(H–1) (i) $g(s)$ is a continuous, monotone, increasing function on $\mathbb{R}$;
(ii) $g(s)\,s > 0$ for $s \neq 0$;
(iii) $M_2 s^2 \leq g(s)\,s \leq M_1 s^2$ for $|s| \geq 1$, for some M_1, M_2, $0 < M_2 \leq M_1$;

(H–2) (i) $f_0(s)$ is a $W^{1,\infty}_{\text{loc}}(\mathbb{R})$, piecewise $C^1(\mathbb{R})$ function, differentiable at $s = 0$;
(ii) $f_0(s)\,s \geq 0$ for $s \in \mathbb{R}$;
(iii) $|f_0'(s)| \leq N(1 + |s|^{k_0 - 1})$, $1 < k_0 < \frac{n}{n-2}$ for $|s| > N$, N large enough, $n \geq 2$;

(H–3) (i) $f_1(s)$ is a continuous function, differentiable at $s = 0$;
(ii) $f_1(s)s \geq 0$ for $s \in \mathbb{R}$;
(iii) $|f_1(s)| \leq M|s|^{k_1} + A|s|$ for $s \in \mathbb{R}$, $k_1 < \frac{n-1}{n-2}$, M, A given constants.

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