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ON ASYMPTOTIC STABILITY FOR LINEAR VISCOELASTIC FLUIDS

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Abstract. The constitutive equation of the classical theory of infinitesimal viscometric fluids is considered, and some restrictions for this equation are found as a direct consequence of thermodynamics principles. These restrictions are proved to be sufficient to prove existence, uniqueness and stability theorems for the boundary-initial history value problem. Subsequently it is shown that the asymptotic stability of the rest state fails when these conditions are not satisfied.

Introduction. This paper presents an evolution problem in a bounded domain Ω for viscoelastic fluids of the kind discussed in [2], [15], [21] and [22][1]. We confine our attention to the classical theory of infinitesimal visco-elasticity and consider isotropic, homogeneous, incompressible fluids. For these materials the linearized constitutive theory states that the *symmetric stress tensor* $\mathbf{T}$ is determined by the *infinitesimal strain history* $\mathbf{E}^t(x,s) = \mathbf{E}(x,t-s)$ through the hereditary law[2]

$$\mathbf{T}(x,t) = -p(x,t)\mathbf{I} + 2\int_0^\infty \mu'(s)[\mathbf{E}^t(x,s) - \mathbf{E}(x,t)]\,ds, \tag{0.1}$$

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[1]A viscoelastic fluid (see Truesdell-Noll [22]) "may remember everything that ever happened to it, yet it cannot recall any one configuration as being physically different from any other except in regard to its mass density", and (see Truesdell [21]) "[a] fluid may have definite memory of all its past experience, [yet] it reacts to those experiences only by comparing them with its present configuration". In other words the stress in a fluid should be unchanged by a change of the reference configuration. Therefore the present configuration is used as reference.

[2]This hereditary law which postulates a linear relationship between the stress and the history is obtained as linearization of the frame-independent constitutive equation of Boltzmann's type

$$\mathbf{T}(x,t) = -p(x,t)\mathbf{I} + 2\int_0^\infty \mu'(s)[\mathbf{C}_t^{-1}(x,t-s) - \mathbf{I}]\,ds,$$

where $\mathbf{C}_t^{-1}$ is a relative strain tensor defined as inverse of the right relative Cauchy-Green tensor (see [15], page 21). Equation (0.1) is not frame indifferent because $\mathbf{E}$ is not properly invariant under changes of frame, thus this theory cannot possibly apply to any material in general finite deformation. However (see [22], page 117), "It is possible... that these [infinitesimal] theories describe the behavior of *some* material for arbitrary finite deformations, even though only in limiting case can they be expected to apply to *all* simple materials with fading memory".