Differential and Integral Equations, Volume 7, Number 4, July 1994, pp. 1145 – 1152.

GROWTH BOUNDS OF SOLUTIONS OF ABSTRACT NONLINEAR DIFFERENTIAL EQUATIONS

G.F. Webb
Mathematics Department, Vanderbilt University, Nashville, TN 37240

Dedicated to the memory of Peter Hess

Abstract. Growth estimates are provided for solutions of the equation $z' = Az + F(z)$, where A is the infinitesimal generator of a C_0-semigroup of operators in a Banach space X and F is a nonlinear operator in X. The rate at which solutions grow to infinity is connected to the rate of growth of the semigroup generated by A and the rate at which $\|F(x)\|/\|x\|$ converges to 0 as $\|x\|$ goes to infinity.

1. Introduction. The objective of this paper is to analyze the way in which the solutions of nonlinear differential equations can grow to infinity. Such equations apply to early stage growth processes during which nonlinearities account for transition or interaction. Late stage growth models may involve different nonlinearities which stabilize or limit growth. The method we use supposes that the nonlinear equation is a perturbation of a linear equation for which growth estimates are known. We employ a variation of constants formula to establish conditions which yield similar growth estimates for the nonlinear problem.

The problem we consider has the form

$$z'(t) = Az(t) + F(z(t)), \quad t \geq 0, \quad z(0) \in X, \tag{1.1}$$

where X is a Banach space, A is the infinitesimal generator of a strongly continuous semigroup $T(t), t \geq 0$ of bounded linear operators in X, and F is a nonlinear operator in X. We suppose that

there exist a nonnegative integer n, a nonnegative constant ω, and a positive constant M such that $\limsup_{t\to\infty} e^{-\omega t}|T(t)|/t^n \leq M$, (1.2)

F is bounded on bounded sets, continuous, and there exists $r_0 > 1$ and a nonincreasing function $c : [r_0, \infty) \longrightarrow [0, \infty)$ such that $\|F(z)\| \leq c(\|z\|)\|z\|$ for all $z \in X$ such that $\|z\| \geq r_0$. (1.3)

Received September 1993.
AMS Subject Classifications: 34C20, 34C11.