Differential and Integral Equations, Volume 7, Number 4, July 1994, pp. 1095 – 1108.

ESTIMATES ON THE LOWEST DIMENSION OF INERTIAL MANIFOLDS FOR THE KURAMOTO-SIVASHINSKY EQUATION IN THE GENERAL CASE

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Dedicated to the memory of Peter Hess

Abstract. We derive estimates on the lowest dimension in various Sobolev spaces of inertial manifolds for the Kuramoto-Sivashinsky equation:

$$\frac{\partial u}{\partial t} + \nu D^4 u + D^2 u + uDu = 0$$

for solutions which are periodic with period L. Contrary to earlier results in [3] and other works, there is no requirement on the antisymmetry of the initial data. Our results are: 1. the lowest dimension of inertial manifolds in the Sobolev space H^m is bounded by a universal constant times $\tilde{L}^{0.82m+2.05}$; 2. the lowest dimension of inertial manifolds in L^2 is bounded by a universal constant times $\tilde{L}^{1.64}(\ln \tilde{L})^{0.2}$, where $\tilde{L} = \frac{L}{2\pi\sqrt{\nu}}$.

0. Introduction. An inertial manifold M, which can be viewed as a global center, or center unstable manifold, is a smooth positively invariant finite dimensional manifold that attracts orbits of a dissipative dynamical system at an exponential rate. The present paper is mainly concerned with the lowest dimension of the inertial manifolds of the 1-D Kuramoto-Sivashinsky equation with *general periodic* (non-antisymmetric) boundary condition:

$$\begin{cases} \dfrac{\partial u}{\partial t} + \nu D^4 u + D^2 u + uDu = 0, \quad x \in \Omega, \\ D^j(\frac{-L}{2}, t) = D^j(\frac{L}{2}, t), \quad j = 0, 1, 2, 3; \quad t > 0, \\ \int_\Omega u(x,t)dx = 0, \quad t > 0, \end{cases} \tag{KSE}$$

where $\nu > 0$ is given, L is the period and $\Omega =]\frac{-L}{2}, \frac{L}{2}[$.

This equation has been extensively studied both for its importance as a model equation for pattern formation and combustion and for its non-trivial dynamics. For

Received September 1993.
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AMS Subject Classifications: 35B35, 35B40, 35B45, 35K55, 35Q80, 46N20.