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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF REACTION-DIFFUSION SYSTEMS OF LOTKA-VOLTERRA TYPE

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Dedicated to the memory of Professor P. Hess

1. Introduction. As a mathematical model for the population dynamics of N-species in biology, Lotka [12] and Volterra [17] proposed the ordinary differential system of the form:

$$dv_j/dt = (-e_j + b_j^{-1} \sum_{k=1}^{N} a_{jk} v_k) v_j, \quad j = 1, \ldots, N, \tag{LV}$$

where e_j, $b_j (> 0)$, a_{jk} are given constants; and v_j denotes the biomass of the j-species; and investigated the asymptotic behavior of $v_1, \ldots, v_N$ for large time t.

For $N = 2$, there are extensive literatures on (LV) (or (RD) below), e.g., Copell [5], Henry [7], Rothe [16]. However, for $N \geq 3$, little seems to have been known; see Amann [2, 3], Krikorian [11], Fife-Mimura [6], Friedmann-Tzavars [8], Oshime [14] and others.

In the present paper we consider the reaction-diffusion's version of (LV) of the form:

$$\begin{aligned} &\frac{\partial}{\partial t} u_j = d_j \Delta u_j + u_j f_j(u) \quad (x \in \Omega,\ t > 0) \\ &\frac{\partial}{\partial \nu} u_j \,\Big|_{\partial\Omega} = 0, \quad (t > 0); \quad u_j \,\Big|_{t=0} = \phi_j \quad (j = 1, \ldots, N), \end{aligned} \tag{RD}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, d_j is a positive constant, $\partial/\partial\nu$ denotes the outer normal derivative to $\partial\Omega$, and ϕ_j given smooth non-negative, and not identically zero function satisfying the compatibility condition: $\partial\phi_j/\partial\nu = 0$ on $\partial\Omega$. The purpose of the present paper is to study the asymptotic behavior of solutions of (RD) for large t under some assumptions on f_j.

We suppose that f_j, $j = 1, \ldots, N$, satisfies the following assumptions.

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