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SEMILINEAR PARABOLIC EQUATIONS WITH PREISACH HYSTERESIS

T.D. LITTLE AND R.E. SHOWALTER
Department of Mathematics, The University of Texas, Austin, TX 78712

To the memory of Peter Hess

Abstract. A coupled system consisting of a semilinear parabolic partial differential equation and a family of ordinary differential equations, which is capable of modeling a very general class of hysteresis effects, will be realized as an abstract Cauchy problem. Accretiveness estimates and maximality conditions are established in a product of L^1 spaces for the closure of the operator associated with this problem. Thus, the Cauchy problem corresponding to the closed operator admits a unique integral solution by way of the Crandall-Liggett theory. Special cases of the system include a one-dimensional derivation from Maxwell's equations for a ferromagnetic body under slowly varying field conditions, the Super-Stefan problem, and other partial differential equations with hysteresis terms appearing in the literature.

1. Introduction. We shall consider here the well-posedness of the initial-boundary-value problem for a semilinear (possibly) degenerate parabolic partial differential equation with a hysteresis nonlinearity in the energy. This will include evolution equations of the form of a generalized porous medium equation

$$\frac{\partial}{\partial t}(a(u)+\mathcal{H}(u))-\Delta u=f, \tag{1}$$

in which $a(\cdot)$ is a continuous monotone function and $\mathcal{H}$ is a hysteresis functional; that is, the output $\mathcal{H}(u)$ depends not only on the current value of the input u, but also on the history of the input.

As an elementary but generic example of hysteresis, we mention a functional that arises in the description of the Super-Stefan problem [14]. This functional provides an example of a simple but basic form of hysteresis. The example depends on three parameters, α, β, and ϵ, with $0<\epsilon$, $\alpha<\beta$. Denote by $[x]_+$ and $[x]_-$, respectively, the positive and negative parts of the real number x. The output $w(t)=\mathcal{H}(u(t))$ varies for $t>0$ according to the following:

$$\begin{aligned}&\text{if } u>\beta+\epsilon, \text{ then } w=1;\\&\text{if } u<\alpha-\epsilon, \text{ then } w=-1;\\&\text{if } \alpha-\epsilon<u<\beta+\epsilon, \text{ then } |w|\le 1\end{aligned}$$

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