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ON ELLIPTIC BOUNDARY VALUE PROBLEMS WITH DYNAMIC BOUNDARY CONDITIONS OF PARABOLIC TYPE

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In memory of Peter Hess

Abstract. We consider a second order elliptic operator with boundary conditions containing the first derivative of the trace with respect to time.We are interested in the case when the Dirichlet problem for the elliptic operator is not well posed. Linear and semilinear equations are studied.

1. Introduction. Elliptic problems with dynamic boundary conditions appear in applied mathematics and we mention for this the papers [7] and [8]. See also the recent [3], where also integrodifferential boundary conditions coming from linear viscoelasticity are considered. In his book [10, ch. VI 6], J.L. Lions studies the problem

$$
\begin{aligned}
&A(t)u(t,x) = -\sum_{i=1}^{n}\sum_{j=1}^{n} \partial_i(a_{i,j}(t,x)\partial_j u) = f(t,x), \quad t \in [0,T], x \in \Omega,\\
&\frac{\partial u}{\partial \nu_{A(t)}} - \partial_t(a_1(t,x')u(t,x')) = 0, \quad x' \in \partial\Omega, t \in [0,T],\\
&u(0,x') = u_0(x'), \quad x' \in \partial\Omega, t \in [0,T]
\end{aligned}
$$

where $-\sum_{i=1}^{n}\sum_{j=1}^{n} Re(a_{i,j}(t,x)\xi_i\xi_j) \geq \nu|\xi|^2$ in $\bar{\Omega}$, for any t $\frac{\partial u}{\partial \nu_{A(t)}}$ is the conormal derivative of u on the boundary $\partial\Omega$ of Ω with respect to $A(t)$ and $a_1(t,x') \geq \nu > 0$ for any $(t,x) \in [0,T] \times \partial\Omega$.

The same author treats in his book [11, 1.11.1] the semilinear problem

$$
\begin{aligned}
&\Delta w = 0 \quad \text{in } [0,T]\times\Omega,\\
&\frac{\partial w}{\partial t} + \frac{\partial w}{\partial \nu} + |w|^\rho w = f \quad \text{in } [0,T]\times\partial\Omega,\\
&w(0,x') = w_0(x'), \quad x' \in \partial\Omega.
\end{aligned}
$$

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