Differential and Integral Equations, Volume 7, Number 3, May 1994, pp. 811–821.

DERIVATIVE BLOW-UP AND BEYOND FOR QUASILINEAR PARABOLIC EQUATIONS

MAREK FILA
Department of Mathematical Analysis, Comenius University
Mlynská dolina, 842 15 Bratislava, Slovakia

GARY M. LIEBERMAN
Department of Mathematics, Iowa State University, Ames, Iowa 50011

Dedicated to the memory of Peter Hess

Abstract. L^∞-blow-up of solutions of semilinear parabolic equations has received considerable interest. Several major problems like sufficient conditions for blow-up, the form of the blow-up set, the profile of the solution near a blow-up point or the existence after the blow-up time have been studied. The aim of this paper is to deal with similar questions for a related phenomenon, namely blow-up of the spatial derivative while the solution itself stays bounded. We proceed via the maximum and comparison principles.

1. Introduction. Finite time blow up in the L^∞ norm for solutions of semilinear parabolic equations has received considerable interest. Several major problems like sufficient conditions for blow-up, the form of the blow-up set, the profile of the solutions near a blow-up point, and existence of a (suitable extension of the) solution past the blow-up time have been studied. The aim of this paper is to deal with similar questions for a related phenomenon, namely, blow-up of the spatial derivative while the solution stays bounded.

Specifically, we consider the problem

$$u_t = u_{xx} + f(u_x), \qquad 0 < x < L, \quad t > 0, \tag{1.1}$$

$$u(0,t) = u(L,t) = 0, \qquad t > 0, \tag{1.2}$$

$$u(x,0) = u_0(x), \qquad 0 \le x \le L, \tag{1.3}$$

where $f \in C^2(\mathbb{R})$ satisfies the conditions

$$f(v) > 0 \text{ for all } v, \;\; f'(v) \ge 0 \;\text{ for }\; v \;\text{ large enough}, \tag{1.4}$$

$$\limsup_{v\to\infty} f'(v)/f(v) < \infty, \tag{1.5}$$

$$\int^\infty \frac{v\,dv}{f(v)} < \infty, \tag{1.6}$$

Received August 1993.
AMS Subject Classifications: 35K55, 35K57, 35D99, 35B40.