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PROPERTIES OF THE OPERATOR DOMAINS OF THE FOURTH-ORDER LEGENDRE-TYPE DIFFERENTIAL EXPRESSIONS

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This paper is dedicated to the memory of Professor Peter Hess

Abstract. This paper is concerned with certain properties of the three fourth-order Legendre-type differential expressions. After normalization to the compact interval $[-1, 1]$ of the real line, there are five distinct such differential expressions. There is one of the second order (the classical Legendre differential expression), three expressions of the fourth order (discovered by H.L. Krall in 1938 and 1940), and one of the sixth order (discovered by Littlejohn in 1981). The three fourth-order expressions have a number of interesting properties when considered in the classical integrable-square space on $(-1, 1)$, and in the relevant measure integrable-square spaces on $[-1, 1]$. The paper discusses some of these properties and determines the smoothness conditions satisfied by elements of the maximal domains and the self-adjoint operator domains. These results are related to the orthogonal polynomials generated, firstly in the measure spaces and, secondly, by the fourth-order spectral differential equations linked to the Legendre-type differential expressions.

1. Introduction. The positive and non-negative integers are denoted by $\mathbb{N} = \{1, 2, 3, \ldots, \}$ and $\mathbb{N}_0 = \{0, 1, 2, \ldots, \}$, and the real and complex numbers by $\mathbb{R}$ and $\mathbb{C}$.

With M and N real, non-negative parameters let the monotonic, non-decreasing function $\hat{\mu} : \mathbb{R} \to \mathbb{R}$ be defined by

$$\hat{\mu}(x) = \begin{cases} -1 - M & (x \in (-\infty, -1]) \\ x & (x \in (-1, 1)) \\ 1 + N & (x \in [1, \infty)). \end{cases} \tag{1.1}$$

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