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POSITIVE SOLUTIONS FOR SOME SEMI-POSITONE PROBLEMS VIA BIFURCATION THEORY

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Dedicated to the memory of Peter Hess

Abstract. Bifurcation Theory is used to prove the existence of positive solutions of some classes of semi-positone problems.

1. Introduction. In this paper we deal with the existence of positive solutions of Dirichlet boundary value problems like

$$\begin{cases} -\Delta u = \lambda f(x, u), & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain with smooth boundary $\partial\Omega$, $\lambda > 0$ and $f : \Omega \times \mathbb{R}^+ \to \mathbb{R}$. If $f(x, 0) \geq 0$ then (1) is called a positone problem and has been extensively studied; see, e.g., [3], [10], [11], and the survey [1].

On the contrary we deal here with the so called *semi-positone* (or *non-positone*) problem, when f is such that

(f_1) $f(x, 0) < 0, \quad \forall x \in \Omega.$

Recently some existence results concerning semi-positone problems have been proved; see [4], [6], [7], and [12]. With the exception of [6] (that deals with sub-linear problems and uses sub and super-solutions) the common feature of the papers mentioned above is that they are obtained by means of ODE techniques, such as the shooting method, and hence they handle the case where Ω is an annulus, a ball or a set close to a ball and $f(x, u) = f(u)$.

The main purpose of the present paper is to show that Bifurcation theory can be easily used to study semi-positone problems, like the positone ones. The same abstract setting is employed to handle both asymptotically linear, superlinear as well as sublinear problems on general domains (hence genuine partial differential equations).

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