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ON THE EXACT SOLUTIONS OF THE INTERMEDIATE LONG-WAVE EQUATION

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To the memory of Peter Hess

1. Introduction. The intermediate long-wave equation was introduced by R.I. Joseph [4] as a mathematical model of nonlinear dispersive waves on the interface between two fluids of different positive densities contained at rest in a long channel with a horizontal top and bottom, the lighter fluid forming a horizontal layer above a layer of the same depth of the heavier fluid. When variables have been re-scaled, it is the pseudo-differential operator equation (see [5])

$$\eta_t + 2\eta\eta_x - (N_H\eta)_x + (1/H)\eta_x = 0, \tag{1}$$

where $H > 0$ and the Fourier multiplier operator N_H is given by

$$\widehat{N_H\eta}(k) = (k \coth kH)\widehat{\eta}(k).$$

In common with the classical KdV and Benjamin-Ono equations, between which it was intended to form a model-theoretical bridge [4], equation (1) was found to have a family of exact solitary-wave solutions: namely,

$$\eta(x, t) = \phi_{C,H}(x - Ct),$$

where

$$\phi_{C,H}(x) = \left[\frac{a \sin aH}{\cosh ax + \cos aH}\right], \quad x \in \mathbb{R},$$

for arbitrary $C > 0$ and $H > 0$, and a is the unique solution of the transcendental equation

$$aH \cot aH = (1 - CH), \quad a \in (0, \pi/H).$$

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