Differential and *Integral Equations*, Volume 6, Number 4, July 1993, pp. 949–960.

NULLSPACES, REPRESENTATIONS AND FACTORIZATIONS OF QUASI-DIFFERENTIAL EXPRESSIONS

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(Submitted by: W.N. Everitt)

Abstract. We consider quasi-differential expressions having matrix-valued coefficients on a real interval I. Certain general properties are proved which are new even in the case of scalar expressions. In particular, it is shown in the scalar case (i) that, if two expressions M_1 and M_2 have the same null space, then $M_1 = fM_2$ for some suitable function f and that (ii) if two coefficient matrices $A, B \in Z_n(I)$ generate the same nth order expression then the product of the elements above the leading diagonal is the same for A and B. These properties are used to further prove new results concerning the factorization of scalar expressions.

1. Introduction. It is well-known that classical differential expressions of order n may be represented as a first order system of dimension n. Since early on in this century many authors have considered the generalization of such classical expressions to n-th order quasi-differential expressions which may also be represented as a first order, n-dimensional system. One formulation of scalar quasi-differential expressions was given by Zettl in [14]. A further, broader formulation which includes expressions whose coefficients are themselves matrices was given by Frentzen in [5]. Various general, abstract properties are established in [5] and in particular, a full characterization of all the different matrix representations of the same expression, is found. Those results have proved to be very useful in the ongoing study of quasi-differential expressions (see for example, [6] on commutativity and [11] on factorization).

The aim of the present work is to present two new general properties of quasi-differential expressions, which is achieved in Section 2, then to apply these to obtain some new results concerning the factorization of such expressions, in Section 3 below. The first general property we prove is that there is a quantity associated with any expression which can be computed from any matrix representation of it, but which is the same for all such representations. In the case of the expressions of Zettl [14] this invariant consists of the product of all the entries above the leading

Received February 1992.
AMS Subject Classifications: 34A30.