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WELL-POSEDNESS AND ASYMPTOTICS OF SOLUTIONS FOR A CLASS OF WAVE EQUATIONS WITH A NONLINEAR BOUNDARY STABILIZER

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Abstract. Of concern is the following wave equation with nonlinear dissipation on the boundary:

$$\begin{aligned} &u_{tt}(x,t) = u_{xx}(x,t) \text{ for } (x,t) \in (0,l) \times (0,\infty), \\ &u_x(0,t) \in \beta_0(u_t(0,t)), \\ &-u_x(l,t) \in \beta_1(u_t(l,t)), \\ &u(x,0) = u_0(x), \\ &u_t(x,0) = v_0(x), \end{aligned}$$

where β_0 and β_1 are maximal monotone graphs in $\mathbb{R} \times \mathbb{R}$ each containing the origin. We prove the well-posedness and obtain the associated ω-limit set.

1. Introduction. This paper studies the following wave equation with nonlinear dissipation on the boundary:

$$\begin{cases} u_{tt}(x,t) = u_{xx}(x,t) \text{ for } (x,t) \in (0,l) \times (0,\infty), \\ u_x(0,t) \in \beta_0(u_t(0,t)), \\ -u_x(l,t) \in \beta_1(u_t(l,t)), \\ u(x,0) = u_0(x), \\ u_t(x,0) = v_0(x), \end{cases} \tag{1}$$

where β_0 and β_1 are maximal monotone graphs in $\mathbb{R} \times \mathbb{R}$ such that $0 \in \beta_0(0) \cap \beta_1(0)$. We prove the well-posedness of (1) and obtain the associated ω-limit set. We show that the ω-limit set of (1) only depends on the size of the space domain (l) and on $\beta_0(0)$ and $\beta_1(0)$. In particular, if $\beta_0(0) = \beta_1(0) = 0$, then the solution of (1) with arbitrary initial data will decay to 0 as $t \to \infty$.

To solve (1), we use the nonlinear semigroup theory (which provides unique existence and regularity of the solution [9]) and rewrite (1) as an evolution equation

$$\begin{aligned} &\frac{dw}{dt} = Aw \quad (t > 0), \\ &w(0) = w_0 \end{aligned}$$

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