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UNIFORM PERSISTENCE FOR POPULATION MODELS WITH TIME DELAY USING MULTIPLE LYAPUNOV FUNCTIONS

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Abstract. In this paper we determine sufficient conditions for uniform persistence for multi-species Kolmogorov type models for population interactions with a time-lag in the per capita net growth rates. The method used to obtain the main result involves the construction of multiple Lyapunov functions. Applications to simple food chains are discussed.

1. Introduction. We consider the Kolomogorov type n-population system with time delay

$$\dot{x}_i(t) = x_i(t) f_i(x(t-\tau)) \quad i = 1, \dots, n \tag{1.1}$$

for $t > 0$.

In (1.1), $x(t) = \{x_i(t)\}$, where $x_i(t)$ represents the population density of the ith interacting species at time t, and $\tau > 0$ denotes a constant time delay affecting the density dependence of the per capita net growth rates f_i. We assume that these net growth rates are continuous functions and that system (1.1) is dissipative. Our main result gives additional conditions that yield (uniform) persistence (see [7], for example) for (1.1). Persistence interprets as sustained coexistence or mutual survival for the species represented in (1.1).

The recognition that past history as well as current conditions can influence population dynamics and interactions has motivated the introduction of delays in population growth models. It has been known for awhile that delays can complicate dynamics; equilibria can destabilize and periodic solutions may arise (see, for example, Cooke [2], Cushing [3]). The impact of delays on global properties such as persistence has been more elusive, especially for systems with the generality of (1.1). For Lotka-Volterra type systems some results are known. Gopalsamy [5] determined conditions for global stability in two-species competition interactions. Kuang and Smith [6] have obtained global stability for such systems with delays and diffusion in the presence of strong self-limiting non-delay terms. Wendi and Ma Zhien [8] treat Lotka-Volterra prey-predator differential delay models without the latter restriction, and give conditions for uniform persistence that are independent of delay size, i.e., the conditions are the same as for the ODE model ($\tau = 0$) case. Both [6] and [8] deal with multiple discrete delays; [5] treats distributed delays. In recent work, Cao, Fan and Gard [1] determined sufficient conditions for uniform persistence for two-species Kolmogorov type prey-predator and competition single-delay

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