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MULTIPLICITY RESULTS FOR SEMILINEAR ELLIPTIC EQUATIONS WITH LACK OF COMPACTNESS

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Abstract. In this paper we prove some multiplicity results for the equation

$$\begin{cases} -\Delta u + \lambda u = u^p & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $1 < p < \frac{N+2}{N-2}$, $N \geq 3$, $\lambda > 0$ and $\Omega = \mathbb{R}^N \setminus \bigcup_{i=1}^{k} \overline{\omega}_i$ where ω_i are suitable bounded domains. Moreover the case $p = \frac{N+2}{N-2}$ and Ω bounded is also considered.

Introduction. In this paper we obtain some multiplicity results for positive solutions of the problem

$$\begin{cases} -\Delta u + \lambda u = u^p & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{0.1}$$

where $1 < p < \frac{N+2}{N-2}$, $N \geq 3$, $\lambda > 0$ and $\Omega \subset \mathbb{R}^N$ is an unbounded domain. Problems like (0.1) have been extensively studied when Ω is a bounded domain (see for example [1], [4], [18] and others).

When Ω is an unbounded domain the existence of a solution to (0.1) is a more complicated problem because the embedding $i : \mathrm{H}_0^1(\Omega) \hookrightarrow \mathrm{L}^{p+1}(\Omega)$ (which is compact when Ω is bounded), is not compact when Ω is unbounded. This lack of compactness implies that the functional

$$I_\lambda(u) = \frac{1}{2}\int_\Omega (|\nabla u|^2 + \lambda u^2) - \frac{1}{p+1}\int_\Omega |u^*|^{p+1}$$

(whose critical points are solutions to (0.1)) does not satisfy the Palais-Smale condition, and so the standard variational techniques do not apply (see [11] for some nonexistence results). The first existence results to (0.1) were obtained using symmetry assumptions on Ω to overcome these difficulties (see [5] for the case $\Omega = \mathbb{R}^N$, [8] for other kinds of symmetry).

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