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PERIODIC SOLUTIONS OF FORCED SECOND ORDER EQUATIONS WITH THE OSCILLATORY TIME MAP

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Abstract. In this paper, we consider the periodic solutions of the periodically forced second order equation

$$\frac{d^2x}{dt^2} + f(x)\frac{dx}{dt} + g(x) = p(t, x, \frac{dx}{dt}).$$

The action-angle transformation is used to find conditions such that the above equation can be treated as a perturbation of the equation $\frac{d^2x}{dt^2} + g(x) = 0$. Then, various conditions are given to guarantee the existence and multiplicity of periodic solutions and subharmonics of the considered equation.

1. Introduction. In this paper, we consider the periodic solutions of the periodically forced second order equation

$$\frac{d^2x}{dt^2} + f(x)\frac{dx}{dt} + g(x) = p(t, x, \frac{dx}{dt}), \tag{1.1}$$

where $f, p \in C^0$, $g \in C^1$ and p is 1-least periodic in t. We assume, throughout this paper, the existence and uniqueness of the solution for Cauchy problems associated with (1.1).

Equation (1.1) has been widely investigated for its background in applications and intrinsic nonlinear phenomena (see [12], [11], [23]). If (1.1) is a dissipative system, the existence of periodic solutions follows easily from the uniformly ultimate boundedness of the solutions of the equation. If (1.1) is not a dissipative system, one can study the problem according to the growth of the leading term $g(x)$. When $g(x)$ is superlinear, the Poincaré map of (1.1) has "strong" twist property, and one can use various twist fixed point theorems to prove the existence of periodic solutions. We refer to [24], [6], [5] and [19] in this direction.

When $g(x)$ is semilinear, namely

$$0 < g_* = \liminf_{|x|\to\infty} \frac{g(x)}{x} \le \limsup_{|x|\to\infty} \frac{g(x)}{x} = g^* < \infty,$$

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