Differential and Integral Equations, Volume 6, Number 4, July 1993, pp. 773–792.

ACCRETIVITY RESULTS FOR NONLINEAR SYSTEMS

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Abstract. Sufficient conditions are given for a function $\vec{\varphi} : \mathbb{R}^M \to \mathbb{R}^M$ to be accretive with respect to the norm $|\vec{u}|_1 := \sum_{i=1}^M |u_i|$. Generalizations are also considered. A family of linear elliptic differential operators of second order are m-accretive with respect to that norm. The standard theory of semigroups of nonlinear contractions can then be applied to a class of nonlinear systems of partial differential equations of reaction-diffusion type.

Introduction. This paper deals with nonlinear systems of partial differential equations of the form

$$\frac{d\vec{u}}{dt} + \vec{\varphi}(\vec{u}) + L\vec{u} = \vec{f}, \tag{1}$$

where $\vec{u} : \Omega(\subset \mathbb{R}^N) \to \mathbb{R}^M$ $(N, M \geq 1)$, $\vec{\varphi} : \mathbb{R}^M \to \mathbb{R}^M$, $\vec{f} : \Omega \to \mathbb{R}^M$, and L belongs to a family of linear elliptic differential operators of second order in divergence form.

Sufficient conditions are given for $\vec{\varphi}$ to be accretive with respect to the non-Euclidean norm $|\vec{u}|_1 := \sum_{i=1}^\infty |u_i|$ (with $\vec{u} := (u_1, \dots, u_M) \in \mathbb{R}^M$). For instance, for $M = 3$ we have the case of *antisymmetric pairwise interaction*:

$$\vec{\varphi}(\vec{u}) := \begin{pmatrix} \mu_{11}(u_1) + \mu_{12}(u_1, u_2) + \mu_{13}(u_1, u_2) \\ -\mu_{12}(u_1, u_2) + \mu_{22}(u_2) + \mu_{23}(u_2, u_3) \\ -\mu_{13}(u_1, u_2) - \mu_{23}(u_2, u_3) + \mu_{33}(u_3) \end{pmatrix}, \qquad \forall \vec{u} \in \mathbb{R}^3, \tag{2}$$

where $\mu_{ii} : \mathbb{R} \to \mathbb{R}$ and $\mu_{ij} : \mathbb{R}^2 \to \mathbb{R}$ $(i, j = 1, 2, 3,\ i < j)$. If the μ_{ii}'s are nondecreasing and

$$\begin{cases} \forall \eta \in \mathbb{R},\ \mu_{ij}(\cdot, \eta) & \text{is nondecreasing,} \\ \forall \xi \in \mathbb{R},\ \mu_{ij}(\xi, \cdot) & \text{is nonincreasing,} \end{cases} \tag{3}$$

then $\vec{\varphi}$ is accretive, actually also T-accretive.

If the $-$ signs are replaced by $+$, we get the case of *symmetric pairwise interaction*. Here $\vec{\varphi}$ is accretive if the μ_{ii}'s are nondecreasing and

$$\begin{cases} \forall \eta \in \mathbb{R},\ \mu_{ij}(\cdot, \eta) & \text{is nondecreasing,} \\ \forall \xi \in \mathbb{R},\ \mu_{ij}(\xi, \cdot) & \text{is nondecreasing.} \end{cases} \tag{4}$$

Received for publication May 1992.
AMS Subject Classification: 47B44, 47H20.