Differential and Integral Equations, Volume 6, Number 4, July 1993, pp. 757–771.

BIFURCATION FROM INFINITY AND MULTIPLE SOLUTIONS FOR AN ELLIPTIC SYSTEM

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(Submitted by: Peter Hess)

Abstract. In this paper, we study multiplicity of solutions for a system of semilinear elliptic equations of the form

$$-\Delta u = \lambda u + f(x,u) - v$$
$$-\Delta v = \delta u - \gamma v$$

in some bounded smooth domain in $\mathbb{R}^N$, subject to homogeneous Dirichlet boundary conditions. The parameters δ and γ are positive and satisfy certain relations involving also the first eigenvalue λ_1 of $(-\Delta_0, H^1(\Omega))$. The parameter λ varies in a neighborhood of $\hat{\lambda}_1 := \lambda_1 + \delta/(\gamma+\lambda_1)$. We establish a priori bounds for solutions of the system when λ is an appropriate side of $\hat{\lambda}_1$, depending on the behavior of $f(x,s)$ and $s \to \pm\infty$. These bounds, together with a bifurcation from infinity, gives the multiplicity results.

Introduction. Let Ω be a bounded open subset of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. Consider the semilinear elliptic system depending on the real parameter λ

$$(S_\lambda) \qquad \begin{cases} -\Delta u = \lambda u + f(x,u) - v \\ -\Delta v = \delta u - \gamma v \end{cases} \qquad \text{in } \Omega$$

subject to Dirichlet boundary conditions $u = v = 0$ on $\partial\Omega$; here $f = f(x,s)$ is a real-valued continuous function on $\overline{\Omega} \times \mathbb{R}$ and γ, δ are nonnegative constants. The solutions (u,v) of (S_λ) represent steady-state solutions of reaction-diffusion systems of interest in Biology, see e.g., Rothe [14] and Lazer-McKenna [9].

The non-parametric system S_0 $(\lambda = 0)$ was studied among others by De Figueiredo-Mitidieri [5], who proved the existence of one or even two [pairs (u,v) of] solutions under various assumptions on f, using both monotone iteration techniques and variational methods. In this paper, we study existence and multiplicity of solutions to (S_λ) when λ is near $\hat{\lambda}_1$,

$$\hat{\lambda}_1 := \lambda_1 + \frac{\delta}{\gamma + \lambda_1}$$

Received June 1992.
†Partially supported by the CNPq.
AMS Subject Classification: 35B45, 35B50, 35J50, 35J55.