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OPTIMAL CONTROL FOR NONLINEAR ABSTRACT EVOLUTION SYSTEMS*

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Abstract. For nonlinear controlled abstract evolution systems with Lagrange type cost functional, we have derived the necessary conditions of Pontryagin type for optimal controls of the problem. The basic tools are the Ekeland's variational principle and the spike variation technique for evolution equations.

1. Introduction. In this paper, we are concerned with a controlled nonlinear evolution system of the type

$$\dot{x}(t) = f(t, x(t), u(t)), \quad \text{a.e. } t \in [0, T], \text{ in } X^*. \tag{1.1}$$

Here, we are in the so-called Gelfand triple $X \hookrightarrow H = H^* \hookrightarrow X^*$ with X being a separable reflexive Banach space and H being a Hilbert space, the dual H^* of which is identified with itself (it is referred as the pivot space). In (1.1), $f : [0,T] \times X \times U \to X^*$, with U being a metric space, is a given map. It is important to notice that for any fixed $(t, u) \in [0,T] \times U$, $f(t, \cdot, u)$ maps from X to its dual X^*. Thus, (1.1) covers usual linear, semilinear and quasilinear evolution equations in the framework of Gelfand triple. For example, by taking

$$f(t, x, u) = -Ax + g(t, x, u),$$

with $A : X \to X^*$ being some linear monotone operator, we see that (1.1) is a semilinear system (see [2, 4, 5, 6] for control problems). Also, if we take

$$f(t, x, u) = -A(t, x) + B(t, x)u, \tag{1.2}$$

with proper operator-valued functions $A(\cdot, \cdot)$ and $B(\cdot, \cdot)$, we obtain systems studied in [9, 10]. Hence, we see that (1.1) is very general (although it looks like an ordinary differential equation in finite dimensional space). Similar systems like (1.1) were also studied in [8], which discussed the existence of relaxed optimal controls. We will give some more concrete examples at the end of this paper.

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