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STRONGLY NONLINEAR SECOND-ORDER ODE's WITH UNILATERAL CONDITIONS*

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1. Introduction. In this paper we consider the strongly nonlinear Neumann boundary value problem

$$-(\phi(u'))' = f(t, u(t)) - q(t), \quad u'(a) = u'(b) = 0, \tag{N}$$

where ϕ is an odd increasing homeomorphism on the real line $\mathbb{R}$, and $' = \frac{d}{dt}$. We assume that $f : [a, b] \times \mathbb{R} \to \mathbb{R}$ is a Caratheodory function satisfying the sign condition

$$(f(t, s) - \bar{q})s \geq 0 \tag{1.1}$$

for almost all $t \in (a, b)$ and $|s|$ large, where $\bar{q}$ denotes the mean value of $q \in L^1 := L^1((a, b), \mathbb{R})$. Solutions of (N) are intended in the sense that $u \in C^1([a, b], \mathbb{R})$ with $\phi(u')$ absolutely continuous and satisfying the equation in (N) for almost every $t \in [a, b]$.

We are interested in existence results of the type below the first eigenvalue. In this context we quote here a very recent paper of Shapiro [16], for the PDE case and Dirichlet boundary conditions, where a constant λ_1 which plays the role of the first positive eigenvalue for the strongly nonlinear case is introduced. Then existence of solutions under a nonresonance-like condition below λ_1 is obtained. This constant λ_1 is defined by means the lim inf of a Rayleigh-type quotient.

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