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EXISTENCE AND UNIQUENESS OF COEXISTENCE STATES FOR THE PREDATOR-PREY MODEL WITH DIFFUSION: THE SCALAR CASE

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Abstract. In this paper we solve the problem of the existence and uniqueness of coexistence states for the classical one-dimensional Lotka-Volterra predator-prey model with diffusion.

1. Introduction. In this paper, we shall show the existence and uniqueness of positive solutions in both components (the so called coexistence states) for the model

$$\begin{aligned} -U'' &= \lambda U - AU^2 - BUV, \quad x \in (0,1),\\ -V'' &= \mu V + CUV - DV^2, \quad x \in (0,1), \qquad (1.1)\\ U(0) &= U(1) = V(0) = V(1) = 0, \end{aligned}$$

where A, B, C, D, λ, μ are real numbers such that $A > 0$, $D > 0$, $C \geq 0$ and $B \geq 0$.

Problem (1.1) usually arises in biology and chemistry in modeling the behavior of two interacting species on $(0,1)$. From a biological point of view the real parameters λ and μ describe, if positive, the net birth rates of the species and, if negative, the net death rates. We are assuming logistic growth for both species and that V preys on U.

Under these assumptions the change of variables $u = AU$, $v = DV$ changes (1.1) into

$$\begin{aligned} -u'' &= \lambda u - u^2 - buv, \quad x \in (0,1),\\ -v'' &= \mu v + cuv - v^2, \quad x \in (0,1), \qquad (1.2_b)\\ u(0) &= u(1) = v(0) = v(1) = 0, \end{aligned}$$

where $b = \frac{B}{D}$ and $c = \frac{C}{A}$. Throughout this paper we shall restrict our attention to (1.2).

In references [1–3], [5–6] and [10–17] were obtained some existence and uniqueness results for (1.2) in general bounded domains Ω of $\mathbf{R}^N$ with smooth enough boundary.

The characterization of the set of values of (λ,μ) for which (1.2) has some coexistence state is well known (see [1–2], [5–6] and [17]). Such a characterization

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