EXISTENCE AND UNIQUENESS OF COEXISTENCE STATES FOR THE PREDATOR-PREY MODEL WITH DIFFUSION: THE SCALAR CASE

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Abstract. In this paper we solve the problem of the existence and uniqueness of coexistence states for the classical one-dimensional Lotka-Volterra predator-prey model with diffusion.

1. Introduction. In this paper, we shall show the existence and uniqueness of positive solutions in both components (the so called coexistence states) for the model

\[-U'' = \lambda U - AU^2 - BUV, \quad x \in (0,1),\]
\[-V'' = \mu V + CUV - DV^2, \quad x \in (0,1),\]
\[U(0) = U(1) = V(0) = V(1) = 0,\]  

(1.1)

where $A$, $B$, $C$, $D$, $\lambda$, $\mu$ are real numbers such that $A > 0$, $D > 0$, $C \geq 0$ and $B \geq 0$.

Problem (1.1) usually arises in biology and chemistry in modeling the behavior of two interacting species on $(0,1)$. From a biological point of view the real parameters $\lambda$ and $\mu$ describe, if positive, the net birth rates of the species and, if negative, the net death rates. We are assuming logistic growth for both species and that $V$ preys on $U$.

Under these assumptions the change of variables $u = AU$, $v = DV$ changes (1.1) into

\[-u'' = \lambda u - u^2 - buv, \quad x \in (0,1),\]
\[-v'' = \mu v + cuv - v^2, \quad x \in (0,1),\]
\[u(0) = u(1) = v(0) = v(1) = 0,\]  

where $b = \frac{B}{D}$ and $c = \frac{C}{A}$. Throughout this paper we shall restrict our attention to (1.2).

In references [1-3], [5-6] and [10-17] were obtained some existence and uniqueness results for (1.2) in general bounded domains $\Omega$ of $\mathbb{R}^N$ with smooth enough boundary.

The characterization of the set of values of $(\lambda, \mu)$ for which (1.2) has some coexistence state is well known (see [1-2], [5-6] and [17]). Such a characterization

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