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DISTRIBUTED SYSTEMS OF PDE IN HILBERT SPACE*

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Abstract. We present a system of two nonlinear evolution equations and a corresponding approximating system which provide a common framework for studying *distributed microstructure models* and a variety of other models for transport and diffusion in heterogeneous media. Existence and uniqueness are demonstrated using semigroup methods, and solutions to the approximating system are shown to converge strongly to the solution of the limiting system. In the microstructure case, new results are obtained, and additional PDE examples are provided to show that in general, certain hypotheses cannot be removed.

1. Introduction. Various models of mass transport and diffusion through heterogeneous media lead to systems of partial differential equations which share a rather general structure. Among the most successful of these are the *dual porosity* models, and these vary considerably in complexity. The simplest of these are the *parallel flow* models consisting of two independent flow equations coupled by an exchange proportional to difference in pressures in the two components. These include the parabolic system

$$\frac{\partial}{\partial t}(u_1) - \vec{\nabla} \cdot A(\vec{\nabla} u_1) + a_0(u_1) + \frac{1}{\varepsilon}(u_1 - u_2) = f_1 \tag{1.1.a}$$

$$\frac{\partial}{\partial t}(u_2) - \vec{\nabla} \cdot B(\vec{\nabla} u_2) + b_0(u_1) + \frac{1}{\varepsilon}(u_2 - u_1) = f_2. \tag{1.1.b}$$

and the *first-order kinetic* models, e.g., the above with $B = 0$. In order to include the geometric effects of an intricate interface between the components, one uses *distributed microstructure models.* Here a single macroscopic flow equation is coupled to a continuum of flow equations, one at each point in space representing adsorption and internal flow or reaction in a corresponding adsorption site. An example is the following system. The macroscopic flow is given by

$$\frac{\partial}{\partial t}\big(u(x,t)\big) - \vec{\nabla} \cdot A(x, \vec{\nabla} u) + q(x,t) = f(x,t) \ , \qquad x \in \Omega \ , \tag{1.2.a}$$

where $q(x,t)$ is the exchange term representing the flow into the cell Ω_x located at x. The flow within the local cell Ω_x is described in the micro-scale variable y by

$$\frac{\partial}{\partial t}\big(U(x,y,t)\big) - \vec{\nabla}_y \cdot B(x,y,\vec{\nabla}_y U) = F(x,y,t) \ , \qquad y \in \Omega_x \ . \tag{1.2.b}$$

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