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DECAY OF SOLUTIONS TO NONLINEAR, DISPERSIVE WAVE EQUATIONS

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Abstract. The asymptotic behavior of solutions to the initial-value problem for the generalized Korteweg-de Vries-Burgers equation

$$u_t + u_x + u^p u_x - \nu u_{xx} + u_{xxx} = 0$$

and the generalized regularized long-wave-Burgers equation

$$u_t + u_x + u^p u_x - \nu u_{xx} - u_{xxt} = 0$$

is studied for $\nu > 0$ and $p \geq 2$. The decay rate of solutions of these equations is that exhibited by solutions of the linearized equation in which the nonlinear term is simply dropped.

1. Introduction. This paper is concerned with the decay of solutions of the damped wave equations

$$u_t + u_x + u^p u_x - \nu u_{xx} + u_{xxx} = 0, \quad (x \in \mathbb{R},\ t > 0) \tag{1.1}$$

and

$$u_t + u_x + u^p u_x - \nu u_{xx} - u_{xxt} = 0, \quad (x \in \mathbb{R},\ t > 0) \tag{1.2}$$

with initial-value conditions

$$u(x,0) = f(x), \quad (x \in \mathbb{R}). \tag{1.3}$$

In the above equations, $u = u(x,t)$ is a real-valued function of the two real variables x and t, subscripts adorning u connote partial differentiation, ν is a positive number and p is a positive integer.

Such equations arise as mathematical models for the unidirectional propagation of nonlinear, dispersive, long waves. In this sort of application, u is typically an amplitude or a velocity, x is proportional to distance in the direction of propagation and t is proportional to elapsed time. Important special cases of (1.1) and (1.2) are the well known Korteweg-de Vries equation (KdV equation)

$$u_t + u_x + uu_x + u_{xxx} = 0, \tag{1.4}$$

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