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ON A CLASS OF NONLINEAR HIGH ORDER VARIATIONAL INEQUALITY SYSTEMS

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Abstract. In this paper, we discuss a class of nonlinear high-order variational inequality systems. We obtain the existence of a solution and some regularity results. We introduce a suitable test function, belonging to the closed convex set of the variational inequality, with which we establish the estimate of regularity of the solution by the maximal function technique. We also discuss the behavior of the solution near the free boundary by obtaining some local estimates.

1. Introduction. In this paper, we consider the existence and regularity of a class of nonlinear high-order variational inequality system as follows:

$$u \in \mathbb{K}, \quad \langle Au, v-u\rangle \geq 0 \quad \forall v \in \mathbb{K}, \tag{1}$$

where $\mathbb{K}$ is a closed convex, nonempty set in an L-vector-valued Banach space $\mathbf{B} = [W^{M,p}(\Omega)]^L$, $p > 1$, where Ω is a bounded $C^{M-1,1}$ domain in $\mathbb{R}^N$, $M, N, L \in \mathbb{N}$, $N \geq 2$. We take

$$\mathbb{K} = \big\{v \in \mathbf{B} : (\tfrac{d}{d\nu})^i v = 0 \text{ on } \partial\Omega,\ i = 0,1,\dots, M-1,\ v \geq \Psi \ \text{ in } \Omega\big\},$$

for $\Psi \in [W_0^{M,p}(\Omega)]^L \cap [L_\infty(\Omega)]^L$, where $v = (v_1, \dots, v_L)$ and $v \geq 0$ means each component $v_l \geq 0$, $l = 1, 2, \dots, L$, $\frac{d}{d\nu}$ denotes the outside normal derivative at $\partial\Omega$, and

$$\langle Au, v\rangle = \int_\Omega \sum_{l=1}^{L} \sum_{|\gamma|\leq M} A_\gamma^l(x, D^M u)\Big(\frac{\partial}{\partial x}\Big)^\gamma v_l \, dx \quad u, v \in \mathbf{B}, \tag{2}$$

where $D^M u$ denotes the set of derivatives

$$\Big\{\Big(\frac{\partial}{\partial x}\Big)^\alpha u_l\Big\}_{\substack{l=1,\dots,L;\\ |\alpha|\leq M}} = \Big\{\frac{\partial^{|\alpha|} u_l}{\partial x_1^{\alpha_1} \dots \partial x_N^{\alpha_N}} : l = 1, \dots, L,\ \alpha = (\alpha_1, \dots, \alpha_N),$$

$$\alpha_i \in \mathbb{N}_0,\ |\alpha| = \sum_{i=1}^{N} \alpha_i \leq M\Big\},$$

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