

MULTIPLICITY OF PRINCIPAL BOUNCE TRAJECTORIES WITH PRESCRIBED MINIMAL PERIOD ON RIEMANNIAN MANIFOLDS

FABIO GIANNONI

Istituto di Matematiche Applicate, Università di Pisa
Via Bonanno Pisano 25/b, 56126 Pisa, Italy

(Submitted by: Jean Mawhin)

Abstract. Let (\mathcal{M}, g) be a smooth Riemann manifold and Ω an open bounded subset of \mathcal{M} such that $\Omega \cup \partial\Omega$ is homeomorphic to the unitary ball of \mathbb{R}^n and $\partial\Omega$ is locally convex. Let $V \in C^2(\mathcal{M}, \mathbb{R})$ and $T > 0$. Using the theory of the subordinated classes, under suitable assumptions on V and T we prove the existence of at least n periodic solutions of the ordinary differential equations

$$D_t \dot{x}(t) + \text{grad } V(x(t)) = 0$$

having minimal period $2T$ and bouncing orthogonally against the boundary of Ω .

1. Introduction. On a Riemannian manifold (\mathcal{M}, g) of class C^3 consider the ordinary differential equation

$$D_t \dot{x}(t) + \text{grad } V(x(t)) = 0, \tag{1.1}$$

where \dot{x} is the derivative of the curve $x(t)$, $D_t \dot{x}$ the covariant derivative of \dot{x} and $\text{grad } V$ the gradient of V with respect to the Riemannian structure g .

Several studies have been recently made about the existence of solutions of (1.1) bouncing against the boundary of a given domain (cf. [5, 7] for the one-dimensional Cauchy problem, [6, 20] for the Cauchy problem on a smooth Riemannian manifold, [8, 13, 19] for the illumination problem in a domain of \mathbb{R}^n and [1, 12, 19] about the periodic problem in a domain of \mathbb{R}^n).

In [12] has been studied the multiplicity problem for periodic bounce solutions of (1.1) having prescribed minimal period, moving in a convex bounded subset Ω of \mathbb{R}^n and bouncing against the boundary of Ω , hitting it orthogonally. These special periodic bounce solutions have been called principal bounce trajectories.

Moreover the nice and important result proved in [4] can be interpreted as a bounce result. There the existence of at least n orthogonal geodesic chords was proved whenever Ω is an open subset of a Riemannian manifold such that $\Omega \cup \partial\Omega$ is homeomorphic to the unitary ball of \mathbb{R}^n and $\partial\Omega$ is locally convex.

In this paper a multiplicity result for principal bounce trajectories (in presence of a conservative field) on a Riemannian manifold is proved. It generalizes the result

Received for publication July 1992.

AMS Subject Classification: 58E05, 59F22.