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# ON THE CAUCHY PROBLEM FOR A NONLOCAL NONLINEAR SCHRÖDINGER EQUATION OCCURRING IN PLASMA PHYSICS

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**Abstract.** In this work, our goal is to study the Cauchy problem for some generalization of the following system.

$$i\phi_t + \Delta\phi = -\text{div}\,(|\nabla\psi|^2\nabla\psi), \quad \Delta\psi = \phi.$$

In particular, we shall construct solutions in the energy space associated to this system. We give some sufficient conditions on the initial data which ensure that the solutions are global, but we show that in some cases, finite time blow up occurs.

## 1. Introduction.

**1.1. Derivation of the equations.** In a nonlinear plasma, one can observe two types of motions (see S.L. Musher, A.M. Rubenchick and V.E. Zakharov [12]): high-frequency electron oscillations and low-frequency ones involving ions. We confine ourselves to the consideration of long wave oscillations. This makes it possible to consider low frequency motions as quasi-neutral. The interaction of high frequency oscillations will be neglected, which allows us to describe them using linearized hydrodynamical equations for an electron gas. Using the Maxwell's equations, we obtain

$$\frac{1}{c^2}\Big(\frac{\partial^2}{\partial t^2} + \omega_p^2\Big)\mathbf{E} + \mathbf{curl}\;\mathbf{curl}\,\mathbf{E} - \frac{3v_{T_e}^2}{c^2}\nabla\,\text{div}\,\mathbf{E} + \frac{\omega_p^2}{c^2 n_0}\delta_n\mathbf{E} = 0,$$

where $\mathbf{E}$ is the electric field, $n_0$ the density of electron at the state of rest and $\omega_p$ the pulsation of the plasma. We consider oscillations with a frequency close to that of the plasma and an electric field of the form

$$\mathbf{E} = e^{i\omega_p t}\tilde{\mathbf{E}},$$

with $\frac{\partial\tilde{\mathbf{E}}}{\partial t} \ll \omega_p\tilde{\mathbf{E}}$. Neglecting the second derivative, we get

$$2i\omega_p\frac{\partial\tilde{\mathbf{E}}}{\partial t} + c^2\mathbf{curl}\;\mathbf{curl}\;\tilde{\mathbf{E}} - 3v_{T_e}^2\nabla\text{div}\,\tilde{\mathbf{E}} + \frac{\omega_p^2}{n_0}\delta_n\tilde{\mathbf{E}} = 0.$$

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