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THE PERIOD FUNCTION OF A HAMILTONIAN QUADRATIC SYSTEM*

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Abstract. It is shown that, for a planar Hamiltonian quadratic system with a center, the period of the associated periodic orbits is a strictly increasing function of the energy.

1. Introduction.

Let

$$dx/dt = H_y, \qquad dy/dt = -H_x \tag{1}$$

be a planar Hamiltonian system with a center, which for definiteness we assume located at the origin. The origin is surrounded by a continuous family of periodic orbits. Each periodic orbit in this continuous family lies on an energy level set $H(x,y) = h$ and may be denoted by $\gamma(h)$, since it is uniquely determined by h. The *period function* $T(h)$ is the (least) period of $\gamma(h)$.

The dependence of the period on the energy has been extensively studied. On the one hand there is interest in *isochronous* systems, for which $T(h)$ is a constant. On the other hand, in studying the perturbation of periodic orbits by Melnikov's method (see, e.g., Guckenheimer and Holmes [8]) it is assumed that the derivative $T'(h)$ is nonzero, so that $T(h)$ is a strictly monotonic function.

Conditions which ensure that $T'(h) \neq 0$ have been given for specific types of Hamiltonian system by several authors. Schaaf [10] has considered systems of the form

$$dx/dt = g(y), \qquad dy/dt = -f(x),$$

and the special case

$$d^2x/dt^2 + f(x) = 0 \tag{2}$$

has received particular attention. It is known that the period function for a center of (2) is monotonic if $\int_0^x f(\xi)\, d\xi / f^2(x)$ is convex (Chicone [1]), or if f is a polynomial

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