Differential and Integral Equations, Volume 6, Number 6, November 1993, pp. 1349–1356.

DIFFUSION IN NONHOMOGENEOUS ENVIRONMENT WITH PASSIVE DIFFUSION INTERFACE CONDITIONS

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(Submitted by: Klaus Schmitt)

Abstract. Boundary value problems of the type

$$p_i^{-1}(x)\{p_i(x)g_i(x,y_i)y_i'\}' = f_i(x,y_i); \quad i-1 \le x \le i, i = 1,2$$

$$y_1'(0) = h_1(y_1(0)); \quad y_2(2) = h_2(y_2'(2))$$

$$\alpha_1 p_1(1)g_1(1,y_1(1))y_1'(1) = \alpha_2 y_2(1) - y_1(1)$$

$$p_1(1)g_1(1,y_1(1))y_1'(1) = \beta p_2(1)g_2(1,y_2(1))y_2'(1)$$

are used to model steady–state diffusion of a specimen in a two patch nonhomogeneous environment. It is assumed that the interaction between the two patches is by passive diffusion. Existence and uniqueness results are obtained by matching solutions for each patch so that the interface conditions are satisfied. Corresponding results are discussed when either both or one of the boundary conditions are replaced by $y_1(0) = h_3(y_1'(0))$ or $y_2'(2) = h_4(y_2(2))$ respectively.

1. Introduction.

We consider boundary value problems of the type

$$p_i^{-1}(x)\{p_i(x)g_i(x,y_i)y_i'\}' = f_i(x,y_i); \quad i-1 \le x \le i; i = 1,2 \tag{1.1}$$

$$y_1'(0) = h_1(y_1(0)) \tag{1.2}$$

$$y_2(2) = h_2(y_2'(2)) \tag{1.3}$$

$$\alpha_1 p_1(1)g_1(1,y_1(1))y_1'(1) = \alpha_2 y_2(1) - y_1(1) \tag{1.4}$$

$$p_1(1)g_1(1,y_1(1))y_1'(1) = \beta p_2(1)g_2(1,y_2(1))y_2'(1) \tag{1.5}$$

where primes denote differentiation with respect to x. It is assumed throughout that $f_i(x,y)$, $g_i(x,y)$, $p_i(x)$ and $h_i(y)$ are continuous for $i-1 \le x \le i$, $-\infty < y < \infty$, with $\frac{\partial}{\partial x}(p_i(x)g_i(x,y))$ and $\frac{\partial}{\partial y}(p_i(x)g_i(x,y))$ existing on $[i-1,i] \times \mathbb{R}$. Further, it is assumed that $\alpha_1 \ge 0, \alpha_2, \beta > 0$, $p_1(x) > 0$ on $(0,1]$, $p_2(x) > 0$ on $[1,2]$ and $g_i(x,y) > 0$ on $[i-1,i] \times \mathbb{R}$. The singular case $p_1(0) = 0$ is allowed.

Received November 1992, in revised form March 1993.
AMS Subject Classifications: 34B15.