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# UNIQUENESS OF SOLUTIONS TO THE CAUCHY PROBLEM FOR $u_t - u\Delta u + \gamma|\nabla u|^2 = 0$

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Dedicated to Professor Riichi Iino on the occasion of his 70th birthday

**Abstract.** We prove the uniqueness of weak solutions and of viscosity solutions of the Cauchy problem for $u_t - u\Delta u + \gamma|\nabla u|^2 = 0$ in $\mathbb{R}^N \times (0,T)$ in the class of semi-superharmonic functions. We also discuss the equivalence of these two notions of generalized solutions.

**1. Introduction.** Let $\gamma$ be a nonnegative number and $u_0$ a nonnegative function on $\mathbb{R}^N$. We consider the Cauchy problem

$$\text{(CP)} \quad \begin{cases} u_t - u\Delta u + \gamma|\nabla u|^2 = 0 & (x,t) \in \mathbb{R}^N \times (0,T), \quad (1.1) \\ u(x,0) = u_0(x) & \text{for } x \in \mathbb{R}^N. \quad (1.2) \end{cases}$$

We will be concerned with nonnegative solutions of (CP). For $u \geq 0$, equation (1.1) is of a degenerate parabolic type, and hence we cannot expect in general for (1.1) to have a classical solution. We thus need the notion of generalized solutions and shall adopt two kinds of generalized solutions in our study of (CP). One of these kinds is the class of weak solutions which will be defined in Section 2 and is similar to the one studied by Bertsch, Dal Passo and Ughi [1], [2]. The other is the class of viscosity solutions which has been introduced by Lions [10] and Crandall and Lions [5]. (The reader is warned not to confuse the use of the term "viscosity solutions". The term "viscosity solutions" is also used to indicate the solutions constructed by the method of vanishing viscosity in the literature, for instance, in [3]. This is not the one that we use. We only use the term here to indicate the one introduced by Crandall and Lions.)

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