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NEUMANN PROBLEM FOR SINGULAR DEGENERATE PARABOLIC EQUATIONS

Yoshikazu Giga†
Department of Mathematics, Hokkaido University, Sapporo 060, Japan

Moto-Hiko Sato‡
Department of Mathematics, Tokyo Metropolitan University,
Minami Ohsawa, Tokyo 192-03, Japan

In the memory of Professor Peter Hess

Abstract. We prove a comparison theorem for viscosity solutions of singular degenerate parabolic equations with the Neumann boundary condition on a domain not necessarily convex. Our result applies to various level set equations including the Neumann problem for the mean curvature flow equations where every level set of solutions moves by its mean curvature and perpendicularly intersects the boundary of the domain.

1. Introduction. This paper continues our investigation [19, 11] on the Neumann problem for singular degenerate parabolic equations. A typical example is

$$u_t - |\nabla u| \text{ div } (\nabla u/|\nabla u|) = 0 \quad \text{on} \quad (0,\infty) \times \Omega, \tag{1.1}$$

$$\partial u/\partial \nu = 0 \quad \text{on} \quad (0,\infty) \times \partial\Omega, \tag{1.2}$$

where Ω is a bounded domain in $\mathbb{R}^n$ and $\partial/\partial\nu$ denotes the outer normal derivative on $\partial\Omega$. As explained in [3, 8] the first equation asserts each level set of u is evolving by its mean curvature in Ω at least formally. The boundary condition formally says that each level set of u intersects perpendicularly with the boundary $\partial\Omega$. We often call (1.1) the level set equation of the motion by mean curvature.

The level set equation (1.1) with $\Omega = \mathbb{R}^n$ provides a new notion of generalized mean curvature motion of hypersurface in $\mathbb{R}^n$ as studied by [3], [8] and others. The generalized motion is given as the zero level set of u as defined by [3] and [8] (see also [4] for corrections). One of key ingredients of their theory is to establish the comparison principle for viscosity solutions of (1.1) (see [6] for the theory of viscosity solutions).

In [19] the second author adapted the level set approach by [3] and [8] to geometric evolutions of hypersurfaces (in Ω) intersecting with $\partial\Omega$ perpendicularly. He established the comparison principle for the Neumann problem including (1.1)–(1.2) as a typical example. However, his method needs the convexity of Ω.

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