Differential and Integral Equations, Volume 6, Number 6, November 1993, pp. 1201–1215.

ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF A NONSTANDARD SECOND ORDER DIFFERENTIAL EQUATION

WILLIAM A. HARRIS, JR.
Department of Mathematics, University of Southern California, Los Angeles, CA 90089-1113

PATRIZIA PUCCI
Department of Mathematics, Universita' degli Studi
Via Vanvitelli 1, 06123 Perugia, Italy

JAMES SERRIN
School of Mathematics, University of Minnesota, Minneapolis, MN 55455

Abstract. The asymptotic behavior as $t \to \infty$ of solutions of the linear differential equation

$$y'' + 2b\,t^{\alpha-1}y' + c\,t^{2\beta-2}y = 0,$$

is determined when α, β and b, c are real constants. The (α, β) plane is divided into sectors by rays from the origin which represent lines of change in the asymptotic representation of solutions.

1. Introduction.

We are concerned with the asymptotic behavior at infinity of solutions of the linear differential equation

$$y'' + 2b\,t^{\alpha-1}y' + c\,t^{2\beta-2}y = 0, \tag{1.1}$$

where α, $\beta \in \mathbb{R}$ and b, c are non–zero real constants. Such problems arise for example in the study of radial solutions of the linear elliptic equation $\Delta u + c|x|^{2\beta-2}u = 0$ in n dimensions, in which case $u(r) = u(|x|)$ is a solution of

$$u''(r) + \frac{n-1}{r}\,u'(r) + c\,r^{2\beta-2}u(r) = 0, \qquad r > 0.$$

Another case of importance is that of a damped harmonic oscillator, when $\beta = c = 1$ and $b > 0$. More generally we may think of (1.1) as a linear differential equation with a highly irregular singular point at ∞.

We shall show that the rather unexpected results indicated in [4] for the special case $\beta = 1$ have the following more general manifestation: the (α, β) plane is divided into sectors by the set of rays from the origin $(0, 0)$ given by

$$\begin{cases} \beta < 0, & \alpha = 0 \\ \beta = s_N\alpha, & \alpha > 0, \quad N = 0, 1, \dots, \quad s_N = 1 - 1/2(N+1) \\ \beta = \alpha, & \alpha > 0 \\ \beta = \alpha/s_N, & \alpha > 0, \quad N = 0, 1, \dots \\ \beta > 0, & \alpha = 0 \\ \beta = 0, & \alpha < 0, \end{cases} \tag{1.2}$$

Received for publication June 1993.
AMS Subject Classifications: 34E05, 34A30.