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EXPONENTIAL STABILIZATION FOR A KIRCHHOFF PLATE WITH BOUNDARY NONLINEARITIES

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Abstract. This paper considers the problem of global uniform stability of a Kirchhoff plate where "energy conserving" nonlinearities appear in the boundary conditions. After implementing appropriate feedback control laws, we use energy methods combined with a specialized compactness-uniqueness argument to achieve the desired stability.

1. Introduction. In this work, we examine the problem of global exponential stabilization for a nonlinearly perturbed Kirchhoff plate for a class of boundary nonlinearities. The system we consider describes the motion of a thin plate which is clamped along one portion of the boundary while being controlled through a "free edge" boundary condition on the remainder of the boundary. We ask the question: "Can we, by appropriately selecting a boundary control, produce an exponential decay rate for the energy of the system?" In contrast to the problem of local stabilization (see [2]), we consider a system where the size of the initial data is unrestricted. By relaxing the restrictions on our initial data, we may expect that the restrictions on the nonlinear term to become more stringent. Here we will prove that for the class of nonlinearities which are "energy conserving" we obtain global stability. By this, we mean that our nonlinear term does not cause the energy of the system to increase as time progresses. Note that since these nonlinearities are not dissipative, they will not contribute to the stability of the system.

1.1. Statement of problem and main result. Let Ω be an open domain in $\mathbb{R}^2$ with smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$, where Γ_i are relatively open, $\bar{\Gamma}_0 \cap \bar{\Gamma}_1 = \emptyset$ and $\Gamma_0 \neq \emptyset$. We consider the following plate equation:

$$\begin{aligned}
& w_{tt} + \Delta^2 w = 0 && \text{in } Q = \Omega \times (0,T) && (a)\\
& w(0,\cdot) = w_0, \quad w_t(0,\cdot) = w_1 && \text{in } \Omega && (b)\\
& w = \frac{\partial}{\partial \nu} w = 0 && \text{on } \Sigma_0 = \Gamma_0 \times (0,T) && (c)\\
& \Delta w + (1-\mu) B_1 w = 0 && \text{on } \Sigma_1 = \Gamma_1 \times (0,T) && (d)\\
& \frac{\partial}{\partial \nu} \Delta w + (1-\mu) B_2 w = w_t + f(w) && \text{on } \Sigma_1 && (e)
\end{aligned} \qquad (1.1)$$

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