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SUPERCONVEXITY OF THE EVOLUTION OPERATOR AND PARABOLIC EIGENVALUE PROBLEMS ON $\mathbb{R}^n$

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1. Introduction. The purpose of this paper is to investigate the stability of the zero solution of the equation

$$\partial_t u - k(t)\Delta u = \lambda m(x,t)u \qquad \text{in } \mathbb{R}^N \times (0,\infty) \tag{1.1}$$

as the parameter λ varies over $\mathbb{R}^+$. Here we assume that the *diffusion coefficient* $k\colon \mathbb{R} \to \mathbb{R}$ is a smooth and strictly positive T-periodic function ($T > 0$ a fixed number) and the *weight function* $m\colon \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is smooth and T-periodic in the second argument (for the precise smoothness conditions consult Section 6). Furthermore, we shall assume that m changes sign and that

$$m(x,t) \leq -c < 0 \qquad \text{for all } (x,t) \in \mathbb{R}^N \times \mathbb{R} \text{ with } |x| \geq R_0 \tag{1.2}$$

holds with some suitable constants $c, R_0 > 0$. We remark that by suitable rescaling of time we could assume without loss of generality that $k \equiv 1$.

Stability shall be understood as stability with respect to the L_∞-norm and initial values in $C_0(\mathbb{R}^N)$, the space of continuous functions vanishing at infinity. More precisely, we shall interpret (1.1) as an abstract evolution equation in the Banach space $X_0 := (C_0(\mathbb{R}^N), \|\cdot\|_\infty)$. This is accomplished by setting

$$X_1 := D(A) := \{u \in X_0 : \Delta u \in X_0\},$$

$$Au := -\Delta u \qquad \text{for } u \in X_1 \text{ and}$$

$$M(t)u := m(\cdot\,,t)u(\cdot) \qquad \text{for } u \in X_0 \text{ and } T \in \mathbb{R}.$$

It is a well-known fact that $-A$ is the infinitesimal generator of a strongly continuous analytic semigroup on X_0 (see e.g., [10]). Moreover, $M\colon \mathbb{R} \to \mathcal{L}(X_0)$ is smooth and T-periodic. We may rewrite (1.1) as

$$\dot{u} + k(t)Au = \lambda M(t)u \qquad \text{for } 0 < t \leq T, \tag{1.3}$$

which is a linear evolution equation in X_0. The assumptions we have made are such that for each $u_0 \in X_0$, (1.3) admits a unique solution $t \mapsto u(t;u_0)$ satisfying the initial condition

$$u(0) = u_0. \tag{1.4}$$

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