Differential and Integral Equations, Volume 7, Number 1, January 1994, pp. 217–234.

BIFURCATION FOR STRONGLY INDEFINITE FUNCTIONALS AND A LIAPUNOV TYPE THEOREM FOR HAMILTONIAN SYSTEMS

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(Submitted by: Jean Mawhin)

Abstract. Suppose that the linear Hamiltonian system $\dot{z} = JAz$ has a nonconstant periodic solution. It is known that the perturbed system $(*)$ $\dot{z} = J\nabla H(z)$, where $\nabla H(z) = Az + o(|z|)$ as $z \to 0$, may have no periodic solutions other than 0. In this paper sufficient conditions for the existence of small periodic solutions of $(*)$ are given. A related bifurcation problem for the nonautonomous Hamiltonian system $\dot{z} = \lambda J\nabla H(z,t)$ and for the elliptic system $-\Delta u = \lambda F_v'(x,u,v)$, $-\Delta v = \lambda F_u'(x,u,v)$ is also studied. The proofs use an infinite dimensional Morse theory for strongly indefinite functionals.

1. Introduction.

Let A be a symmetric $2N \times 2N$ matrix and let

$$J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

be the standard simplectic matrix in $\mathbb{R}^{2N}$. It is well known (cf. Section 3) that the linear Hamiltonian system

$$\dot{z} = JAz \tag{1}$$

has nonconstant periodic solutions of period $2\pi/\beta_0$ if $i\beta_0$ is an eigenvalue of JA. One of the main purposes of this paper is to investigate the existence of small periodic solutions of period close to $2\pi/\beta_0$ for the system

$$\dot{z} = J\nabla H(z), \tag{2}$$

where $\nabla H(z) = Az + o(|z|)$ as $z \to 0$.

Consider first the second order system

$$-\ddot{x} = \nabla F(x) \tag{3}$$

in $\mathbb{R}^N$. The following result is due to Berger.

Received July 1992, in revised form March 1993.

†Supported in part by the Swedish Natural Science Research Council.

AMS Subject Classifications: 34C25, 35J65, 58E05, 58E07.