Differential and Integral Equations, Volume 7, Number 1, January 1994, pp. 205–216.

DUAL SEMIGROUPS FOR DELAY DIFFERENTIAL EQUATIONS WITH UNBOUNDED OPERATORS ACTING ON THE DELAYS

Miklavž Mastinšek
EPF — Univerza v Mariboru, Razlagova 14, Maribor 62000, Slovenia

(Submitted by: Franz Kappel)

1. Introduction. The purpose of this paper is to study delay differential equations (DDE) of the form

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Ax(t-h) + \int_{-h}^{0} a(s)Ax(t+s)\,ds, \text{ for } t \geq 0 \\ &x(0) = \phi^0, \quad x(s) = \phi^1(s) \quad \text{a.e. on } [-h, 0), \end{aligned} \tag{1.1}$$

where A is the infinitesimal generator of an analytic semigroup on a Hilbert space X and $a(\cdot)$ is a scalar-valued integrable function. Equations of this type were considered by Di Blasio, Kunisch and Sinestrari in [4], [5]. They have developed a state space theory with initial values $\phi = (\phi^0, \phi^1)$ from the product space $Z = V \times L^2(-h, 0; D(A))$, where V is a real interpolation space between $D(A)$ and X ($D(A)$ denotes the domain of A endowed with the graph norm).

Bernier, Delfour and Manitius in their study of DDE in R^n (see [1], [3]) have shown that the so-called structural operators can be employed to describe the influence of the delay part in (1.1) on the evolution of the trajectories as well as characterize the adjoint semigroup of the solution semigroup of (1.1). Their results have been recently proved for the case of infinite dimensional spaces, where bounded and unbounded operators act in the delayed part of the equation; see [6], [9], [10], [11], [14] for example. In [11] Nakagiri studied structural properties of functional differential equations in Banach spaces with bounded operators acting on the delays. In [14] Tanabe considered DDE (1.1) for the case where the operator A is defined by a sesquilinear form and $a(\cdot)$ is Hölder's continuous.

In this paper a method allowing weaker assumptions on (1.1) is introduced; i.e., A is assumed to be the generator of an arbitrary analytic semigroup and $a(\cdot)$ is a square integrable function. The paper is organized as follows. In Section 2 the solution semigroup $T(t)$ of the DDE (1.1) is given. By replacing the operator A acting on the delay part of (1.1) by its Yosida-type approximation A_λ, the approximative delay differential equation (ADDE) is obtained. The solution semigroup $T_\lambda(t)$ of ADDE is defined for every $\lambda > 0$ and strong convergence $T_\lambda(t)\phi \to T(t)\phi$ is shown for $\phi \in Z$. In Section 3 the transposed semigroups $T^T(t)$ and $T_\lambda^T(t)$ are introduced. These semigroups are defined as solution semigroups of dual equations, which are

Received in revised form August 1992.
AMS Subject Classification: 34K30, 35R10.