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BIFURCATION FOR SOLUTIONS OF PRESCRIBED MEAN CURVATURE PROBLEMS ON $\mathbb{R}^n$

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Abstract. This article deals with a prescribed mean curvature problem for which the linearization about the trivial solution zero has continuous spectrum. A global bifurcation result is established showing that infinitely many continua of radially symmetric solutions, which are distinguished by nodal properties, emanate from the line of trivial solutions at the lowest point of the continuous spectrum. Nevertheless, the existence of nontrivial solutions is ensured only if the mean curvature is small.

0. Introduction. We study existence and bifurcation of the solutions of the boundary value problem

$$-\mathrm{div}\Big(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\Big) = \lambda u - F(x,u)u, \quad x \in \mathbb{R}^N, \tag{0.1}$$

$$u(x) \to 0 \quad \text{as} \quad |x| \to \infty, \tag{0.2}$$

where λ is an eigenvalue parameter and F satisfies certain assumptions detailed in Section 1.

Equation (0.1) has received a great deal of attention in the analysis of capillary surfaces [13, 16]. In that context, however, $F \equiv 0$ and the interest was focused on singular solutions. In view of the importance of the minimal surface equation and partly inspired by studies on the analogous problem for the Laplace operator, several authors recently dealt with regular solutions of (0.1), (0.2) with $F(x,u)$ behaving like $-|u|^\sigma$, $\sigma > 0$. It has been shown that there is no positive solution if

$$\lambda < -[2(\sigma+2)/\sigma]^{\sigma/(\sigma+2)} \tag{0.3}$$

or

$$\lambda < 0, \quad \sigma \geq \frac{4}{N-2} \quad \text{when} \quad N > 2. \tag{0.4}$$

It is also known that a positive solution does not exist if $\lambda > 0$, or $\lambda = 0$ and $\sigma < \frac{4}{N-2}$. However, for $\lambda = 0$ and $\sigma \geq \frac{4}{N-2}$, Ni and Serrin [28] showed that there are infinitely many positive solutions. In a recent work, Peletier and Serrin [29] proved the existence of positive solutions if $\lambda < 0$ and $|\lambda|$ small, which confirms an earlier numerical study by Evers and Levine. Though the critical exponent $\frac{4}{N-2}$ has

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