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HOMOGENIZATION OF SOME QUASILINEAR PROBLEMS FOR STRATIFIED MEDIA WITH LOW AND HIGH CONDUCTIVITIES

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Abstract. We are concerned with the homogenization of quasilinear elliptic problems $(\mathcal{P}^\varepsilon)$ on structures which are stratified in some direction (say x_1), when the coefficients belong to L^∞ but are neither uniformly bounded from above or from below with respect to ε. The general framework of this study covers as a subcase the case of linear diffusion equations with conductivity matrices $A^\varepsilon(x_1)$ which are diagonal and, as well as their inverses, are not uniformly elliptic with respect to ε. We give assumptions on the coefficients –convergence in the sense of measures and a weakened uniform ellipticity condition– which imply the convergence of the problems $(\mathcal{P}^\varepsilon)$ to a problem of the same form with L^∞–coefficients too.

0. Introduction. Let $\Omega =]0,1[\times\Omega'$ be a cylindrical domain in $\mathbb{R}^N$ representing a structure which is stratified in the x_1-direction; we consider quasilinear elliptic problems $(\mathcal{P}^\varepsilon)$ –well-posed in the Sobolev space $W^{1,p}(\Omega)$– of the form

$$\begin{cases} -\frac{\partial}{\partial x_1} g_1\big(x, a_1^\varepsilon(x_1)\frac{\partial u^\varepsilon}{\partial x_1}\big) - \sum_{i=2}^N a_i^\varepsilon(x_1)^{p-1}\frac{\partial}{\partial x_i} g_i\big(x', \frac{\partial u^\varepsilon}{\partial x_i}\big) = f \ \text{ in } \Omega, \\ +\text{boundary conditions } L, \end{cases} \qquad (\mathcal{P}^\varepsilon)$$

where $x = (x_1, x')$, $x' = (x_2, \dots, x_N)$ and L stands for Dirichlet or mixed Dirichlet-Neumann boundary conditions. It is supposed that the functions g_i have suitable growth and convexity properties, and that the coefficients a_i^ε which only depend on x_1, that is, the meaning of stratification in this paper, satisfy $C_i^\varepsilon \ge a_i^\varepsilon(x_1) \ge c_i^\varepsilon > 0$ almost everywhere for $i = 1, \dots, N$.

This paper is devoted to the homogenization of such problems in the case where the functions a_i^ε and $\frac{1}{a_i^\varepsilon}$ may not be bounded in $L^\infty(0,1)$ as ε tends to zero.

Instead of fully describing the class of problems $(\mathcal{P}^\varepsilon)$ that are concerned, let us introduce the chosen framework by examples of growing generality. The simplest

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