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WEAK INTERIOR SECOND ORDER DERIVATIVE ESTIMATES FOR DEGENERATE NONLINEAR ELLIPTIC EQUATIONS

N.V. Krylov

School of Mathematics, University of Minnesota, Minneapolis, MN, 55455

(Submitted by: Luis A. Caffarelli)

Abstract. We are dealing with the Dirichlet problem for elliptic Bellman equations with underlying linear operators. Under some mild assumptions we prove that second order derivatives of a solution can be estimated in the interior of the domain via estimates on the boundary of the function itself and its derivatives up to the second order, the maximum of the second order normal derivative entering the estimate with arbitrary small coefficient.

1. Introduction. The general theory of uniformly nondegenerate nonlinear concave or convex in second order derivatives elliptic and parabolic equations of second order is rather well developed. There are books [9], [11] and a great amount of articles addressing these equations. Quite recently, Caffarelli [2] showed how to prove the solvability of these equations not only in classes $C^{2+\alpha}$ but in Sobolev classes as well. Very general results are obtained in [21], [22]. The corresponding theory is developed for the Dirichlet problem as well as for other boundary value problems (see [1], [17], [19] and references in the mentioned works).

The situation with degenerate nonlinear equations is not as good. It is worth noting that so far, even *linear* elliptic degenerate equations have not yet been investigated in all detail, though there are classical works by Kohn-Nirenberg [10], Freidlin [8], and Oleinik-Radkevich [20]. If the equations are degenerate and also nonlinear in second order derivatives, a great amount of work was done if there is some kind of nondegeneracy of the equation on the boundary or in partial cases, the most commonly treated are equations similar to the Monge-Ampère equation. In this regard we mention works [9], [3–6], [11], [18], and [24]. Much attention is also paid to the so called viscosity solutions which allow us to investigate solutions without any hypotheses about their derivatives (cf. [23], [7]). As far as classical solutions of the Dirichlet problem for equations in bounded domains are concerned, as always, the proof of solvability is based on a priori estimates, and the crucial element in this procedure is the estimate of the second order normal derivative on the boundary. Very often (actually, almost always), the last estimate is obtained under the assumption that the equation is at least weakly nondegenerate on the boundary; i.e., the normal second order derivative at a point x_0 on the boundary can be expressed (only) from the equation (considered at this point only) and from the given boundary condition (considered in a neighborhood of x_0) as a function of the first normal derivative, of mixed derivatives and of second order derivatives

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