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ON A SEMILINEAR WEAKLY HYPERBOLIC EQUATION WITH LOGARITHMIC NONLINEARITY

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Abstract. In this paper the equation

$$u_{tt} - a(t)\Delta u = f(u)$$

is investigated, where $a(t)$ is a real analytic, nonnegative function (possibly vanishing), while $f(u)$ is a regular function, increasing as $u \log u$ when $u \to \infty$. The global existence of smooth solutions is proved in one and two space dimensions.

1. Introduction. As it is well known, a semilinear Cauchy problem on $\mathbb{R}_t^+ \times \mathbb{R}_x^n$ of the form

$$u_{tt} - \Delta u = f(u)$$
$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x)$$

need not be globally solvable in C^∞, in the sense that local solutions, which always exist, may blow up in the L^∞ norm after a finite time. In some cases ($n \leq 3$, $f(u) = |u|^{p-1}u$ with $p > 1$ close enough to 1, see [10], [7], [19]) it is even possible to prove that *all* nonzero solutions with compact support in x have a finite lifespan.

Thus, the results of global existence for the above problem require some additional assumption: either 1) small C_0^∞ data, $f(u)$ vanishing at $u = 0$ with order large enough with respect to the space dimension n ([8], [13], [14], [17], [20], [22]), or 2) $f(u) = -|u|^{p-1}u$, and p small enough with respect to n ([2], [11],[21]). In case 2 the existence of a positive conserved energy replaces the smallness of the data. In particular, when there is no restriction on the size of the data, neither on the sign of $f(u)$, then in general, the global existence does not hold for a superlinear $f(u)$ (see in particular [12] where the blow up is proved for $f(u) = u^p$ with p low). However, it is not difficult to show that global solutions exist when the nonlinearity is mild, e.g. $f(u) = |u| \log(1 + |u|)$.

In this paper, we are interested in the question of global existence for a *weakly hyperbolic* Cauchy problem of the form

$$u_{tt} = a(t)\Delta u + f(u) \tag{1}$$
$$u(0,x) = u_0(x), \; u_t(0,x) = u_1(x) \tag{2}$$

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