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SINGULARLY PERTURBED BOUNDARY VALUE PROBLEMS

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Abstract. Singularly perturbed couples of second-order differential equations are studied. The existence of solutions is shown by a Galerkin approximation method together with the Leray-Schauder degree theory. Limit behavior of solutions is investigated for specific problems. Discontinuous versions of such problems are studied as well.

1. Introduction. The purpose of this paper is to study the existence of classical solutions for problems of the type

$$\begin{aligned} -\varepsilon^2 u'' + g_1(u,v,x,\varepsilon) &= 0, \\ v'' + g_2(u,v,x,\varepsilon) &= 0, \\ v(0) = v(1) &= u(0) = u(1) = 0, \\ x \in [0,1], \quad \varepsilon &\neq 0, \end{aligned} \tag{1.1}$$

where g_1, g_2 are continuous functions, ε is a small parameter. The author was stimulated by [5, 6], but we shall be interested in the existence of solutions of (1.1). Our approach is quite different, since we can not apply the implicit function theorem due to the nondifferentiability of g_1, g_2. Instead of this, we follow a method from [3]. Thus we use a Galerkin approximation method together with the Leray-Schauder degree theory. We also study asymptotic behavior of solutions as ε tends to zero for some problems; we show the boundary layer phenomenon of u.

The plan of our paper is as follows. In Section 2, we study (1.1) when the second equation has a general form for v, but is a case of perturbations of two independent equations. We show asymptotic behavior of solutions as $\varepsilon \to 0$ for that case. Knowing that result, in Section 3, we apply it for several examples. In Section 4, we investigate (1.1) partly with a different function g_1 as in the previous sections and partly for the case when g_2 has a specific discontinuity in v. We can show only the existence of solutions for such problems, not their asymptotic behavior.

2. General problems. Let us consider the couple of equations

$$\begin{aligned} &-\varepsilon^2 u'' + f(u,x) + \varepsilon \cdot \phi(u,v,x) = 0, \\ &v = G(v) + \varepsilon \cdot H(u,v), \\ &u(0) = u(1) = 0, \end{aligned} \tag{2.1}$$

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