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NEW A PRIORI ESTIMATES IN GEVREY CLASS OF REGULARITY FOR WEAK SOLUTIONS OF 3D NAVIER-STOKES EQUATIONS

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1. Introduction. In this article we consider the Gevrey class regularity of weak solutions of the periodic Navier-Stokes equations (NSEs) in $\mathbb{R}^3$:

$$\frac{\partial u}{\partial t} - \nu\Delta u + \sum_{i=1}^{3} u_i \frac{\partial u}{\partial x_i} + \operatorname{grad} p = F \quad \text{for } (x,t) \text{ in } \mathbb{R}^3 \times \mathbb{R}_+ , \tag{1.1}$$

$$\operatorname{div} u = 0 \quad \text{for } (x,t) \text{ in } \mathbb{R}^3 \times \mathbb{R}_+ , \tag{1.2}$$

$$u(x,0) = u_0(x) \quad \text{for } x \text{ in } \mathbb{R}^3, \tag{1.3}$$

$$u(\cdot,t) \text{ and } p(\cdot,t) \text{ are } Q\text{-periodic for all } t \text{ in } \mathbb{R}_+, \tag{1.4}$$

where $Q = (0,L)\times(0,L)\times(0,L)$ with $L > 0$. The velocity $u = (u_1, u_2, u_3)$ and the pressure p are the unknown functions defined on $\mathbb{R}^3 \times \mathbb{R}_+$. The kinematic viscosity ν, the density of force f and the initial velocity u_0 are given. For simplicity, we also assume that the average value of u over Q is zero; i.e.,

$$\int_Q u\,dx = 0 , \quad \text{for all } t \text{ in } \mathbb{R}_+ . \tag{1.5}$$

In [2], it is proved that the NSEs (1.1)–(1.5) admit a weak solution u satisfying $u(x,t) \in \mathbb{H}^m = [H^m]^3$ for almost all t in $\mathbb{R}_+$ and for arbitrarily large m. Furthermore , the following estimate is obtained:

$$\int_0^T \frac{\|u(t)\|^2_{\mathbb{H}^{m+1}}}{(1+\|u(t)\|^2_{\mathbb{H}^m})^{\frac{2m}{2m-1}}}\,dt \le K'(1+T) , \tag{1.6}$$

where K' is a constant depending on ν, Q, $\|f\|_{\mathbb{H}^{m-1}}$ and $\|u_0\|_{L^2(Q)}$.

Our goal in this paper is to prove the analogue of the above in the Gevrey class of regularity for weak solutions of the NSEs (1.1)–(1.5). Namely, we prove that the NSEs (1.1)–(1.5) admit a weak solution u satisfying $u(t) \in D(e^{\sigma A^{\alpha/2}})$ (see (2.7) for the definition) with $\sigma > 0$ and $0 < \alpha < 1$ for almost all t in $\mathbb{R}_+$, provided that f

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