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REMARKS ON NONLINEAR SCHRÖDINGER EQUATIONS IN ONE SPACE DIMENSION

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Abstract. We consider the initial value problem for nonlinear Schödinger equations,

$$\begin{cases} i\partial_t u + \frac{1}{2}\partial^2 u = F(u, \partial u, \overline{u}, \partial\overline{u}), & (t,x) \in \mathbb{R}^+ \times \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{†}$$

where $\partial = \partial_x = \partial/\partial x$ and $F : \mathbb{C}^4 \to \mathbb{C}$ is a polynomial having neither constant nor linear terms. Without a smallness condition on the data u_0, it is shown that (†) has a unique local solution in time if u_0 is in $H^{3,0} \cap H^{2,1}$, where $H^{m,s} = \{f \in \mathcal{S}' : \|f\|_{m,s} = \|(1+x^2)^{\frac{s}{2}}(1-\Delta)^{\frac{m}{2}} f\|_2 < \infty\}$, $m, s \in \mathbb{R}$.

1. Introduction. We consider the initial value problem for nonlinear Schrödinger equations,

$$\begin{cases} i\partial_t u + \frac{1}{2}\partial^2 u = F(u, \partial u, \overline{u}, \partial\overline{u}), & (t,x) \in \mathbb{R}^+ \times \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{1.1}$$

where $\partial_t = \partial/\partial t$, $\partial = \partial_x = \partial/\partial x$, u is a complex valued function of $(t,x) \in \mathbb{R}^+ \times \mathbb{R}$, $\overline{u}$ is a complex conjugate of u, and F denotes a complex valued polynomial defined on $\mathbb{C}^4$ such that

$$F(z) = F(z_1, z_2, z_3, z_4) = \sum_{\substack{d \le |\alpha| \le \rho \\ \alpha \in \mathbb{Z}_+^4}} a_\alpha z^\alpha, \tag{1.2}$$

where we have used the standard notation for multi-indices. We assume that there exists $a_{\alpha_0} \neq 0$ for some $\alpha_0 \in \mathbb{Z}_+^4$ with $|\alpha_0| = d$, since, as we see below, the lowest degree d of the polynomial F, rather than the highest degree ρ, determines the character of the problem. The main results in this paper are the following. For notation, see below.

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