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UPPER BOUNDS FOR LIAPUNOV FUNCTIONALS, INTEGRAL LIPSCHITZ CONDITION AND ASYMPTOTIC STABILITY

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Abstract. Consider the functional differential equation with bounded delay

$$X' = F(t, X_t).$$

A classical result of Krasovskii is that if there is a Liapunov functional satisfying

(i) $W_1(|X(t)|) \leq V(t, X_t) \leq W_2(\|X_t\|)$,
(ii) $V'(t, X_t) \leq -W_3(|X(t)|)$, and
(iii) $F(t, \phi)$ is bounded if ϕ is bounded,

then $X = 0$ is uniformly asymptotically stable. In this paper, we investigate the properties of stability, weaken the conditions and examine some examples.

1. Introduction. The objective of this paper is to investigate asymptotic stability of the zero solution of the functional differential equation

$$X'(t) = F(t, X_t), \tag{1}$$

where $X_t(\theta) = X(t+\theta)$ for $-h \leq \theta \leq 0$ and h is a positive constant. Before proceeding we shall set forth some notation and terminology that will be used throughout this paper. Denote by C the space of continuous functions $\phi : [-h, 0] \to \mathbb{R}^n$. For $\phi \in C$ we will use the norm $\|\phi\| := \max_{-h\leq s\leq 0} |\phi(s)|$, where $|\cdot|$ is any convenient norm in $\mathbb{R}^n$. Given $H > 0$, C_H denotes the set of $\phi \in C$ with $\|\phi\| < H$. $X'(t)$ denotes the right-hand derivative at t if it exists and is finite. It is supposed that $F : \mathbb{R}_+ \times C_H \to \mathbb{R}^n$, that F is continuous, and that F takes bounded sets into bounded sets. Here, $\mathbb{R}_+ = [0, \infty)$. Then it is known [2, 4, 5, 8] that for each $t_0 \in \mathbb{R}_+$ and each $\phi \in C_H$ there is at least one solution $X(t_0, \phi)$ defined on an interval $[t_0, t_0 + \alpha)$ and if there is an $H_1 < H$ with $|X(t, t_0, \phi)| \leq H_1$, then $\alpha = \infty$. We also suppose $F(t, 0) \equiv 0$ so that $X = 0$ is a solution of (1), and is called the zero solution.

By means of Liapunov's second method, throughout this paper we work with continuous functionals $V : \mathbb{R}_+ \times C_H \to \mathbb{R}_+$ (called Liapunov functionals) with $V(t, 0) = 0$, whose derivative V' with respect to (1) is defined by

$$V'_{(1)}(t, \phi) = \lim_{\delta\to 0} \sup[V(t+\delta, X_{t+\delta}(t, \phi)) - V(t, \phi)]/\delta.$$

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